Načrtněte graf funkce

(a) $f_1(x,y) = x \cdot y$

(b) $f_2(x,y) = x^2 - y^2 = (x-y)(x+y)$

$\sqrt{2} \cdot x' \quad \sqrt{2} \cdot y'$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$$

a jejich vrstevnice

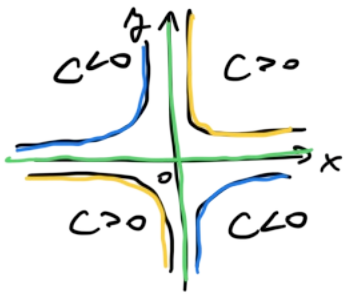
$$f_2 = 2 f_1\left(\frac{x-y}{\sqrt{2}}, \frac{x+y}{\sqrt{2}}\right) = 2 \cdot f_1\left(A(x,y)\right)$$

vrstevnice f_1: $x \cdot y = c$, $c \in \mathbb{R}$

$c > 0$: $y = \frac{c}{x}$

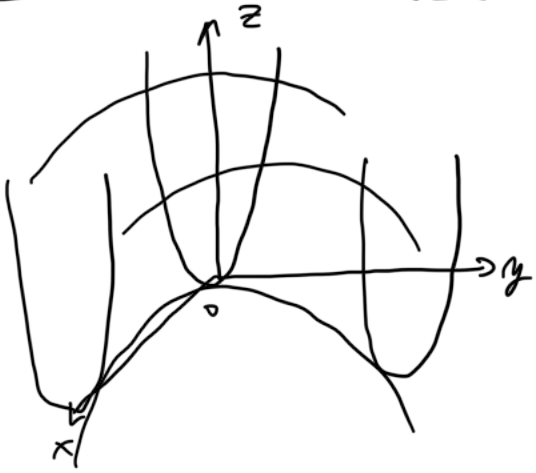
$c < 0$: $y = \frac{c}{x}$

$C = 0$: $x = 0 \mid y = 0$



vrstevnice f_2:

$$f_2(x,y) = x^2 - y^2$$



sedlová plocha

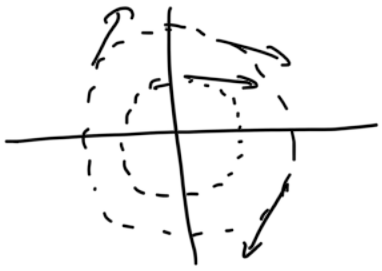
hyperbolický paraboloid

Načrtněte vektorové pole

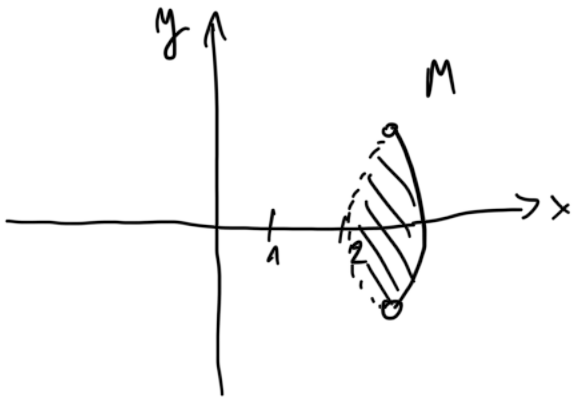
$$\vec{F}(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}, \frac{-x}{\sqrt{x^2+y^2}}\right) = \underbrace{\frac{1}{\|\vec{v}\|}\cdot(y,-x)}_{\text{délka je } 1}$$

$$D(\vec{F}) = \mathbb{R}^2 \setminus \{(0,0)\} \qquad \sqrt{x^2+y^2} = \|\vec{v}\|$$

$$\begin{pmatrix} y \\ -x \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{\text{rotace o } \frac{\pi}{2} \text{ v záporném směru}} \begin{pmatrix} x \\ y \end{pmatrix}$$



Určete vnitřek, hranice a uzávěr množiny $M: (x-1)^2 - y^2 > 1 \wedge (x-2)^2 + y^2 \leq 4$

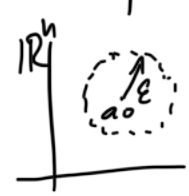


epsilon okolí bodu a:

$$U_\varepsilon(a_0) = \{a \in \mathbb{R}^n \mid \|a - a_0\| < \varepsilon\}$$

$a_0 \in \mathbb{R}^n$

$\varepsilon > 0$



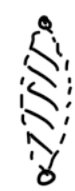
vnitřek množiny $M \subseteq \mathbb{R}^n$ značíme M°

$$a \in M^\circ \Leftrightarrow \exists \varepsilon > 0 \quad \underset{\substack{\rotatebox{90}{$\in$} \\ a}}{U_\varepsilon(a)} \subseteq M$$

$$\Downarrow$$

$$M^\circ \subseteq M$$

vnitřek M (tedy M°):



$\longrightarrow$ oblast bez vnějších čar

$$\Rightarrow (x-1)^2 - y^2 > 1 \wedge (x-2)^2 + y^2 < 4$$

hranice množiny M značíme ∂M:

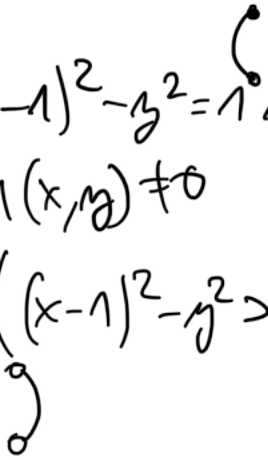
$a \in \partial M \Leftrightarrow \forall \varepsilon > 0 \; U_\varepsilon(a) \cap M \neq \emptyset$

$\wedge \; U_\varepsilon(a) \cap (\mathbb{R}^n \setminus M) \neq 0$

hranice M (bez ∂M): $\left((x-1)^2 - y^2 = 1 \wedge (x-2)^2 + y^2 \leq 4\right)$

$\wedge (x,y) \neq 0$

$\vee \left((x-1)^2 - y^2 > 1 \wedge (x-2)^2 + y^2 = 4\right)$



uzávěr M značíme $\bar{M}$

$a \in \bar{M} \Leftrightarrow \forall \varepsilon > 0 \; U_\varepsilon(a) \cap M \neq \emptyset$

$\partial M \subseteq \bar{M}$, $M \subseteq \bar{M}$

$\bar{M} = \partial M \cup M = \partial M \cup M^\circ$

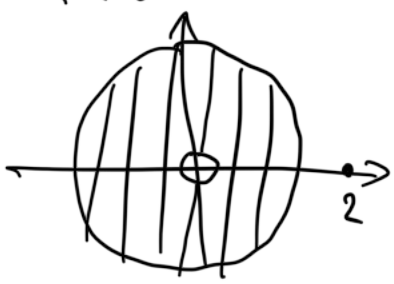
Uzávěr M:

$(x-1)^2 - y^2 \geq 1 \wedge (x-2)^2 + y^2 \leq 4$

$\wedge (x,y) \neq (0,0)$

Najděte vnitřek, hranici a uzávěr a izolované a hromadné hodnoty.

$$M = \{(x,y) \in \mathbb{R}^2 \mid 0 < x^2+y^2 \le 1\} \cup \{(2,0)\}$$



$M^\circ: 0 < x^2+y^2 < 1$

$\partial M: x^2+y^2 = 1$

$V(x,y)=(0,0)$

$V(x,y)=(2,0)$

$\overline{M}: \quad x^2+y^2 \le 1$

$V(x,y)=(2,0)$

$a \in M$ je izolovaný bod M

$\Leftrightarrow \forall \varepsilon > 0 \; U_\varepsilon(a) \cap M = \{a\}$

$\Downarrow$

$P_\varepsilon(a) \cap M = \emptyset$

izolované body M: bod $(2,0)$

$a \in \mathbb{R}^n$ je hromadný bod $M \Leftrightarrow$

$\Leftrightarrow \forall \varepsilon > 0 \; P_\varepsilon(a) \cap M \neq \emptyset$

hromadné body M: $x^2+y^2 \le 1$

množina $M = \mathbb{Q}^2$ v $\mathbb{R}^2$

$M^\circ = \emptyset$

$\partial M = \mathbb{R}^2$

$\overline{M} = M^\circ \cup \partial M = \mathbb{R}^2$

izolované body: $\emptyset$

hromadné body: $\mathbb{R}^2$