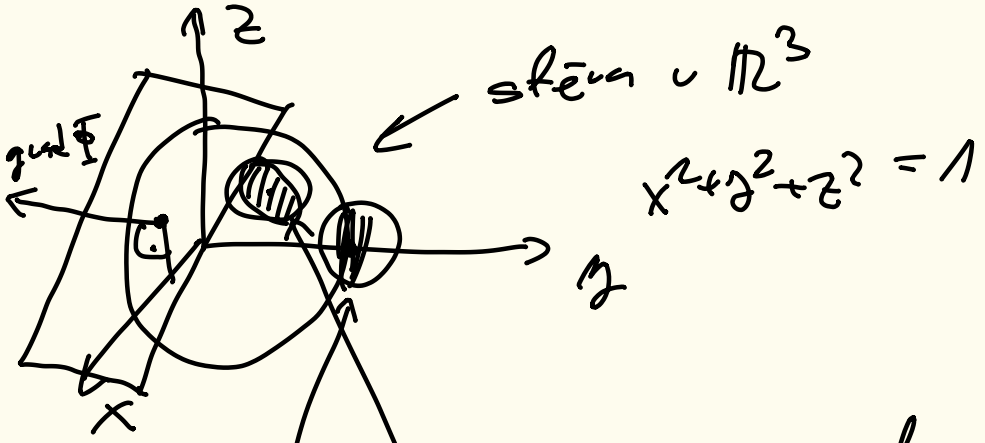


tato část je grafem funkce

$$z = f(x,y) = \sqrt{1-x^2-y^2}$$

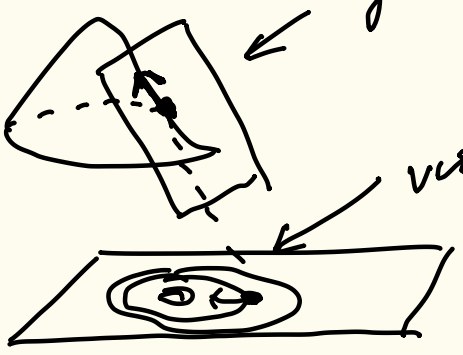
graf funkce, zadaných v proměnných x a z

implicitně definovaná plocha M v $\mathbb{R}^3$ je množina tvaru $M = \{(x,y,z) \in U \mid \Phi(x,y,z)=0\}$, a Φ je spojitě diferencovatelná

kde $\Phi: U \to \mathbb{R}$, $U \subseteq \mathbb{R}^3$ otevřená množina

$$\Phi(x,y,z) = x^2+y^2+z^2-1$$

M je vrstevnice



definujeme jako úhel mezi tečnými rovinami ploch

Najděte úhel, který svírají plochy:

(1) graf fce $f(x,y) = e^{\sin(x\cdot y)}$

(2) plocha H: $(x-1)^2 + \frac{y^2}{2} + (z-3)^2 = 7$

v bodě $A_0 = (0,2,?) = (x_0, y_0, z_0)$

$$\cos\alpha = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\cdot\|\vec{n_2}\|}, \quad \alpha \in \langle 0, \pi/2 \rangle$$

$$z = f(a_0) + \frac{\partial f}{\partial x}(a_0)(x-x_0) + \frac{\partial f}{\partial y}a_0 \cdot (y-y_0)$$

$z_0 = e^{\sin(0\cdot 2)} = e^0 = 1$

$A_0 = (0,2,1)$

$\vec{n_1} = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right) = (2,0,-1)$

$\frac{\partial f}{\partial x} e^{\sin(x\cdot y)} = e^{\sin(xy)} \cdot \cos xy \cdot y \Rightarrow 1\cdot 1\cdot 2 = 2$

$\frac{\partial f}{\partial y} e^{\sin(xy)} = e^{\sin xy} \cdot \cos xy \cdot x \Rightarrow 0$

graf funkce $f = \{(x,y,z) \in D(f)\times\mathbb{R} \mid z = f(x,y)\} =$

$\{ \text{-11-} \mid \Psi(x,y,z) = 0\}$

grad $\Psi(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right)$

$\Psi(x,y,z) = f(x,y) - z$

(2) plocha $M \rightarrow \Phi(x,y,z) = (x-1)^2 + \frac{y^2}{2} + (z-3)^2 - 7$

$\vec{n_2} = \text{grad}\,\Phi = \left(2(x-1),\ y,\ 2(z-3)\right) = (-2, 2, -4)$

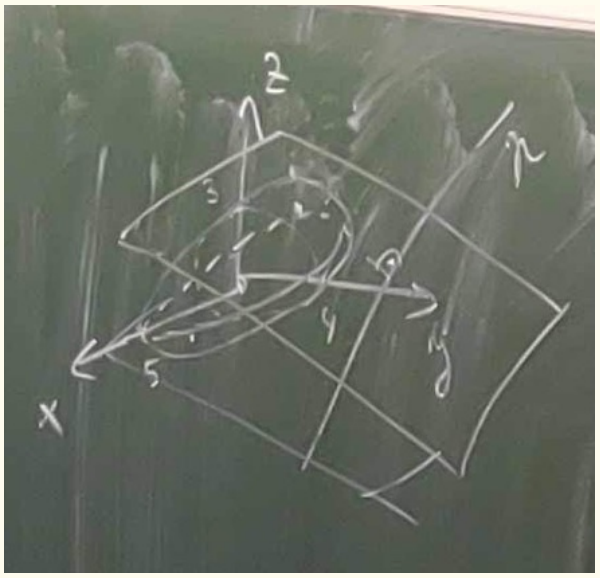
$$\cos\alpha = \frac{|\vec{n_1}\cdot\vec{n_2}|}{\|\vec{n_1}\|\cdot\|\vec{n_2}\|} = \frac{|-4+0-4|}{\dots} = 0$$

$\cos\alpha = 0 \quad \alpha = \pi/2$

Nechť p je přímka v $\mathbb{R}^3$ procházející body $(1,2,3)$, $(2,3,4)$

Najděte rovnici tečné roviny k elipsoidu

$$M: \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = 1$$



aby tato tečná rovina byla kolmá na přímku p

$s_p = (1,1,1)$

$\vec{n} = \text{grad}\,\Phi(x,y,z)$

$\Phi = \frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} - 1$

$\frac{\partial f}{\partial x} = \frac{2x}{25} \qquad \frac{\partial f}{\partial z} = \frac{2z}{9}$

$\frac{\partial f}{\partial y} = \frac{2y}{16}$

směrový vektor roviny je násobkem směrového vektoru přímky p

$\text{grad}\,\Phi(x,y,z) = \lambda \cdot \vec{s_p} = \lambda(1,1,1)$

$\| \quad \left(\frac{2x}{25}, \frac{2y}{16}, \frac{2z}{9}\right)$

chceme splnit tyto rovnice

$$\frac{2x}{25} = \lambda \qquad x = \frac{25}{2}\lambda$$

$$\frac{2}{16}y = \lambda \qquad y = 8\lambda$$

$$\frac{2}{9}z = \lambda \qquad z = \frac{9}{2}\lambda$$

$$\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = 1$$

$$\frac{\left(\frac{25}{2}\lambda\right)^2}{25} + \frac{(8\lambda)^2}{16} + \frac{\left(\frac{9}{2}\lambda\right)^2}{9} = 1 \qquad \lambda = \pm\frac{2}{5}$$

$$(x,y,z) = \pm\left(5, \frac{16}{5}, \frac{9}{5}\right)$$

$s = (1,1,1)$

$x + y + z = d$

$d = \pm\sqrt{2}$

Řetízkové pravidlo

(= aplikace věty o derivaci složeného zobrazení)

Mějme funkci $f = f(u,v) : \mathbb{R}^2 \to \mathbb{R}$, která je spojitě diferencovatelná

Najděte parc. derivaci

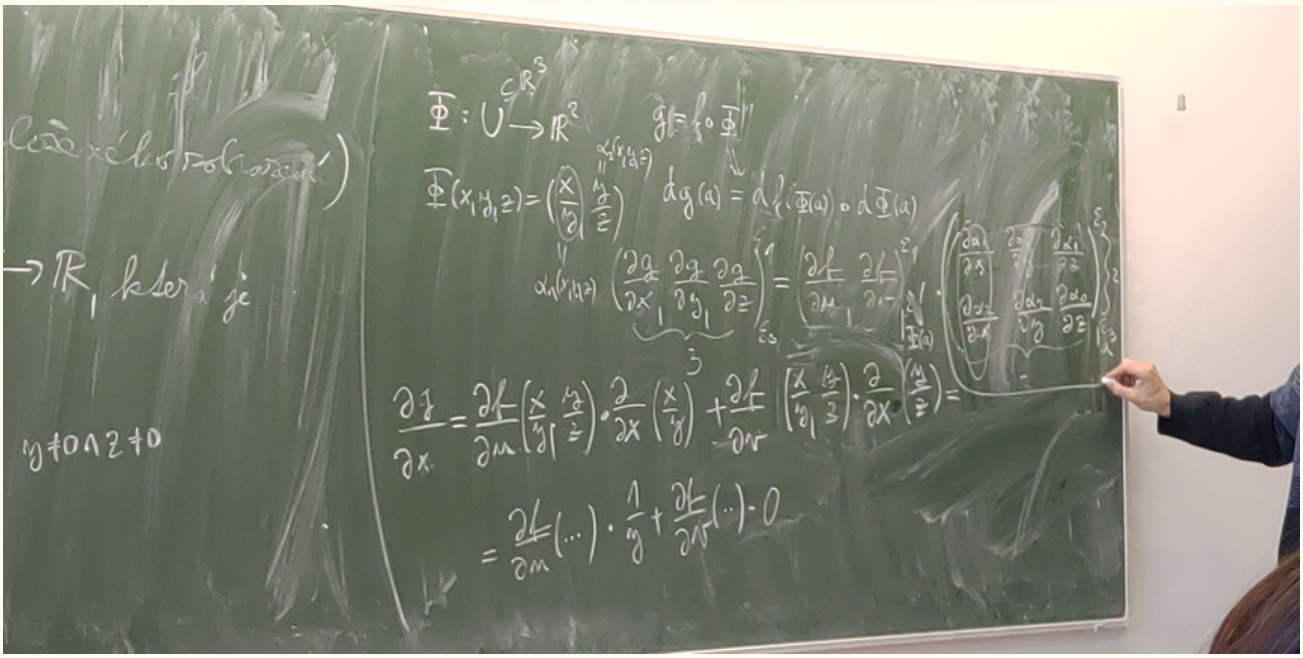
$$\Phi : \overset{\subseteq \mathbb{R}^3}{V} \to \mathbb{R}^2 \qquad g = f \circ \Phi \qquad g(x,y,z) = f\left(\frac{x}{y}, \frac{y}{z}\right)$$

$D(g) : x \neq 0 \wedge z \neq 0$

$$\Phi(x,y,z) = \left(\frac{x}{y}, \frac{y}{z}\right) \qquad dg(a) = df(\Phi(a)) \circ d\Phi(a)$$

$$\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}\right)_{\varepsilon_3}^{\varepsilon_1} = \left(\frac{\partial f}{\partial u}, \frac{\partial f}{\partial v}\right)_{\varepsilon_2}^{\varepsilon_1} \cdot \left(\frac{\partial \alpha_1}{\partial x} \; \frac{\partial \alpha_1}{\partial y} \; \frac{\partial \alpha_2}{\partial z}\right)_{\varepsilon_3}^{\varepsilon_2}$$

3 2 3

$\Phi(q)$

$$(L \circ k)_{B_1}^{B_3} = (L)_{B_2}^{B_3} (k)_{B_1}^{B_2}$$