Spočtěte pomocí polárních souřadnic:

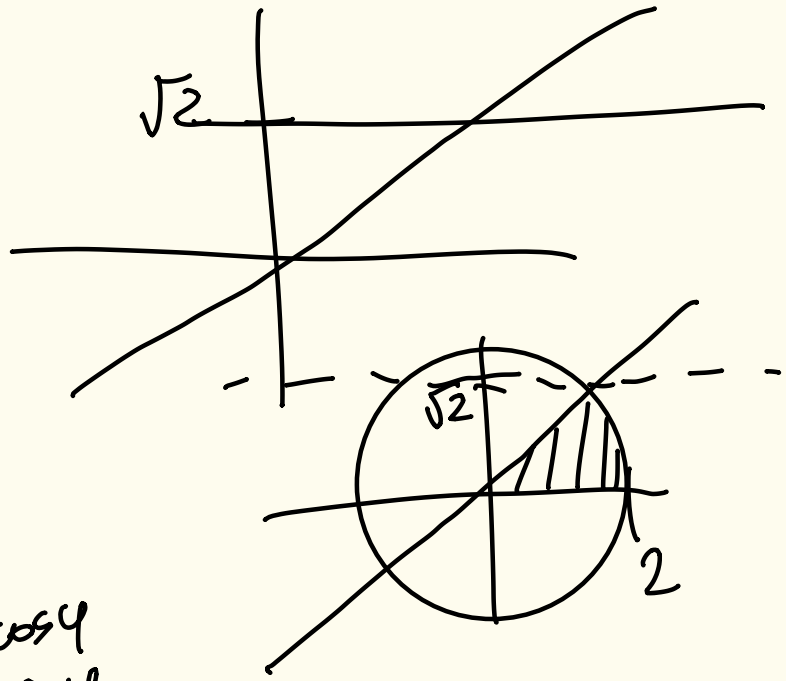
$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\,dx\,dy = \iint_E \frac{1}{1+x^2+y^2}\,dx\,dy$$

E: $y \le x \le \sqrt{4-y^2}$

$0 \le y \le \sqrt{2}$

$x^2 = 4 - y^2$

$x^2 + y^2 = 4$



U: $0 \le r \le 2$

$0 \le \varphi \le \frac{\pi}{4}$

$x = r\cos\varphi$

$y = r\sin\varphi$

jakobián

$$= \iint_U \frac{1}{1+r^2} \cdot r\,dr\,d\varphi$$

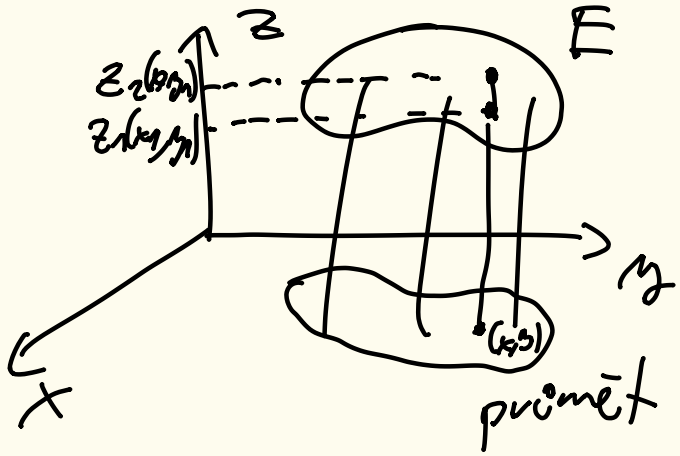
$$= \int_0^2 \int_0^{\frac{\pi}{4}} \frac{r}{1+r^2}\,d\varphi\,dr = \frac{\pi}{4} \cdot \int_0^2 \frac{r}{1+r^2}\,dr =$$

substituce $1+r^2$

$$= \left[\frac{1}{2}\ln(1+r^2)\right]_0^2 = \frac{1}{2}\ln 5$$

$$= \frac{\pi \ln 5}{8}$$

Trojný integrál



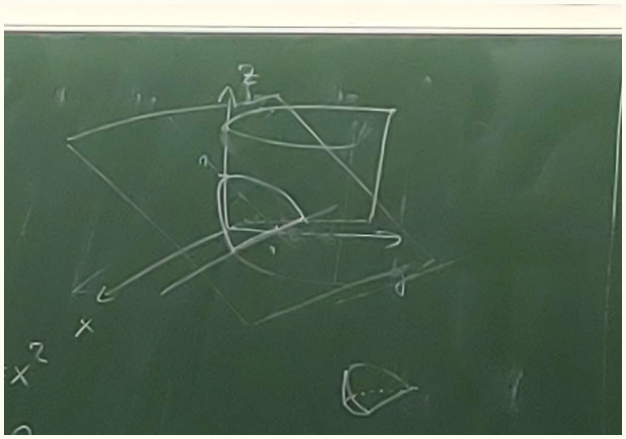
$$\iiint_E f\,dV = \iint_{\pi_{1,2}(E)} \left(\int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\,dz \right) dx\,dy$$

průmět E do roviny xy

Fubiniho věta $\pi_{1,2}(E)$ = průmět E do roviny xy

Spočtěte:

$$\iiint_E z^2\,dV = (*)$$

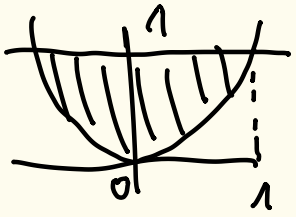


kde E je ohraničena plochami

$y = x^2$
$z = 0$
$y + z = 1$
$\hookrightarrow z = 1 - y$

$E: 0 \le z \le 1-y$

průmět E



$x^2 \le y \le 1$
$-1 \le x \le 1$

$$(*) = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} z^2\,dz\,dy\,dx = \int_{-1}^{1}\int_{x^2}^{1}\left[\frac{z^3}{3}\right]_0^{1-y} dy\,dx = \int_{-1}^{1}\int_{x^2}^{1}\frac{1-y^3}{3}\,dy\,dx$$

$$= -\int_{-1}^{1}\left[\frac{(1-y)^4}{12}\right]_{x^2}^{1} dx = 2\int_0^1 \frac{(1-x^2)^4}{12}\,dx =$$

($\frac{(1-x^2)^4}{12}$ je sudá funkce)

$$h = -x^2 \Big|_{\frac{1}{6}}^{0} \int_0^1 1 - 4x^2 + 6x^4 - 4x^6 + x^8 \, dx =$$

$$\int_0^1 x^n = \frac{1}{n+1}$$

$$= \frac{1}{6}\left(1 - \frac{4}{3} + \frac{6}{5} - \frac{4}{7} + \frac{1}{9}\right)$$