

1. Určete směr největšího růstu funkce $f(x, y) = x^3 e^{y^2} + \sin(x - y)$ v bodě $a_0 = (1, 1)$ a derivaci ve směru $\vec{u} = \left(\frac{1}{2}, \frac{\sqrt{3}}{4}\right)$.

Jiný příklad: Určete tečnou rovinu k ploše $z + 1 = x e^y \cos z$ v bodě $(1, 0, 0)$. Určete také normálu k ploše v daném bodě (tj. přímkou kolmou k tečné rovině a procházející daným bodem).

2. Najděte nejmenší a největší hodnoty funkce

$$f(x, y) = 6 - 4x - 3y$$

za podmínky $x^2 + y^2 = 4y - 2x$. Načrtněte útvar určený touto vazbou.

Jiný příklad: Najděte lokální extrémní funkce

$$f(x, y) = xy(3 - x + y)$$

na otevřené množině

$$M: x > 0.$$

3. Vhodným způsobem integrace spočítejte integrál

$$\iint_E \frac{y^2 x}{x^2 + y^2} dS,$$

kde

$$E: 1 \leq x^2 + y^2 \leq 4 \text{ \& } x \leq 0$$

Oblast E načrtněte.

Jiný příklad: Vypočítejte

$$\iint_E y \, dx \, dy,$$

kde E je trojúhelník s vrcholy $(0, 2)$, $(0, 1)$ a $(1, 2)$.

$$(1a) \nabla f(x, y) =$$

$$\frac{\partial f}{\partial x} = 3x^2 e^{y^2} + \cos(x - y)$$

$$\frac{\partial f}{\partial y} = 2x^3 y e^{y^2} - \cos(x - y)$$

$$\nabla f(1, 1) = (3e + 1, 2e - 1)$$

$$\hat{m} = \frac{\vec{m}}{\|\vec{m}\|} = \left(\frac{\frac{1}{2} \frac{\sqrt{3}}{\frac{\sqrt{7}}{4}}}{\frac{\sqrt{7}}{4}}, \frac{\frac{\sqrt{3}}{4} \frac{\sqrt{3}}{\frac{\sqrt{7}}{4}}}{\frac{\sqrt{7}}{4}} \right) = \left(\frac{2}{\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{7}} \right)$$

$$\|\vec{m}\| = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

→ jednotkový vektor

$$S_m = \hat{m} \cdot \nabla f(1, 1) = \left(\frac{2}{\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{7}} \right) \cdot (3e + 1, 2e - 1)$$

$$= \frac{6e + 2 + 2e\sqrt{3} - \sqrt{3}}{\sqrt{7}} \approx 9,82$$

$$(1b) F(x, y, z) = x e^y \cos z - z$$

$$\frac{\partial F}{\partial x} = e^y \cos z$$

$$\frac{\partial F}{\partial y} = x e^y \cos z$$

$$\frac{\partial F}{\partial z} = -x e^y \sin z - 1$$

normálový vektor tečné roviny
 $\Rightarrow \vec{m} = \nabla F(1, 0, 0) =$

$$\vec{m} = (1, 1, -1)$$

Rovina:

$$x + y - z + d = 0$$

$$\Rightarrow x + y - z - 1 = 0$$

$$1 + d = 0$$

$$d = -1$$

$$(2) \quad f(x, y) = 6 - 4x - 3y \quad f(g) = x^2 + y^2 - 4y + 2x$$

$$\text{za podminky } x^2 + y^2 = 4y - 2x$$

Lagrangeovy multiplikatory: (pokud $\nabla g(a_0) \neq (0, 0)$)

potom $\exists \lambda \in \mathbb{R}$ takové, že

$$\nabla f(a_0) = \lambda \nabla g(a_0)$$

$$\frac{\partial f}{\partial x} = -4 \quad \nabla f(-4, -3) \quad \frac{\partial g}{\partial x} = 2x + 2 \quad \nabla g(2x + 2, 2y - 4)$$

$$\frac{\partial f}{\partial y} = -3 \quad \frac{\partial g}{\partial y} = 2y - 4$$

$kdy \nabla g(x, y) = (0, 0) ?$

$$x = -1$$

$$y = 2$$

$$-4 = \lambda(2x + 2) \quad (1)$$

$$-3 = \lambda(2y - 4) \quad (2)$$

$$0 = x^2 + y^2 - 4y + 2x \quad (3)$$

$$-4 = 2\lambda x + 2\lambda$$

$$2\lambda x = -4 - 2\lambda$$

$$x = \frac{-4 - 2\lambda}{2\lambda}$$

utf soustava?

(2b) Izh. extrémx fce

$$f(x,y) = xy(3-x+y)$$

$$f'(x,y) = 3xy - x^2y + xy^2$$

$$\frac{\partial f}{\partial x} = 3y - 2xy + y^2$$

$$\frac{\partial f}{\partial y} = 3x - x^2 + 2xy$$

$$\nabla f(x,y) = (3y - 2xy + y^2, 3x - x^2 + 2xy)$$

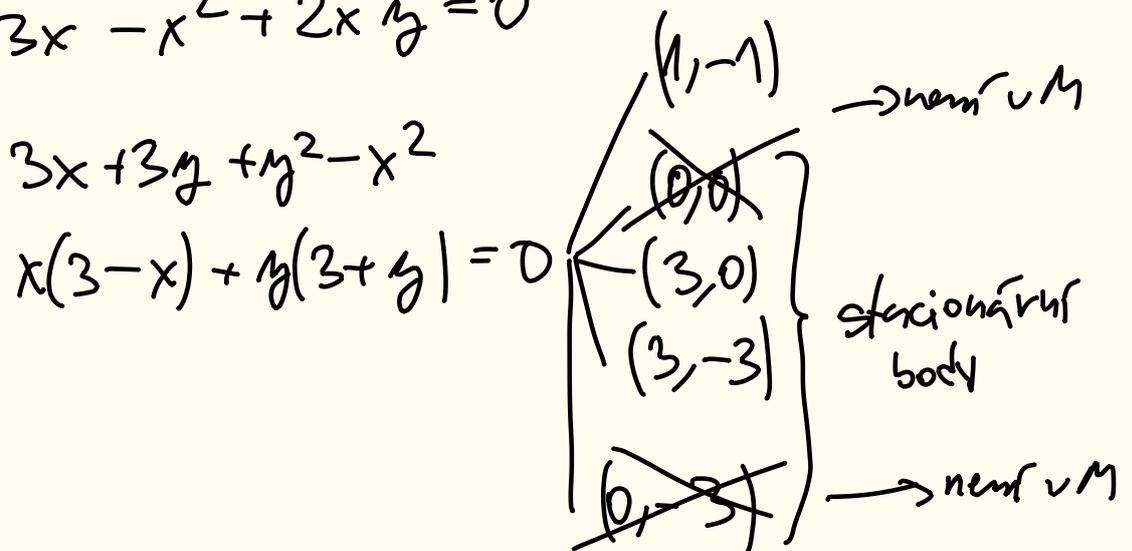
Izh. extrém poloh se $\nabla f(x,y) = (0,0)$

tedy nutně: $3y - 2xy + y^2 = 0$

$$3x - x^2 + 2xy = 0$$

$$3x + 3y + y^2 - x^2$$

$$x(3-x) + y(3+y) = 0$$



Hessova matice $H_f(x,y) = \begin{pmatrix} -2y & 3-2x+2y \\ 3-2x+2y & 2x \end{pmatrix}$

3-2-2

$$\text{Bod } (1, -1) = H_f(1, -1) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad D_1 = 2 > 0$$

$$D_2 = 4 - 1 = 3 > 0$$

pozitivně definitní / lok. min

$$\text{Bod } (3, -3) : H_f(3, -3) = \begin{pmatrix} 6 & -9 \\ -9 & 6 \end{pmatrix} \quad D_1 = 6 > 0$$

$$D_2 = 6 \cdot 6 - (-9 \cdot -9) < 0$$

negativně definitní
maximum

(1.) směr největšího rostu $f(x, y) = x^3 e^{y^2} + \sin(x - y)$
v bode $a_0 = (1, 1)$ a derivaci ve směru $\vec{n} = (\frac{1}{2}, \frac{\sqrt{3}}{4})$

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

$$\frac{\partial f}{\partial x} = 3x^2 e^{y^2} + \cos(x - y)$$

$$\frac{\partial f}{\partial y} = 2x^3 e^{y^2} y - \cos(x - y)$$

$$\nabla f(x, y) = \left(3x^2 e^{y^2} + \cos(x - y), 2x^3 e^{y^2} y - \cos(x - y) \right)$$

v bode: $\nabla f(1, 1) = (3e+1, 2e-1)$

směr největšího rostu: $\vec{n} = \frac{\nabla f(1, 1)}{\|\nabla f(1, 1)\|} = \underline{\underline{\left(\frac{3e+1}{\alpha}, \frac{2e-1}{\alpha} \right)}}$

$$\|\nabla f(1, 1)\| = \sqrt{(3e+1)^2 + (2e-1)^2} = \alpha$$

Derivace ve směru $\vec{n} = (1/2, \sqrt{3}/4)$ je $\frac{\partial f}{\partial \vec{n}}(1,1) =$

$$= (3e+1, 2e-1) \left(\frac{1}{2}, \frac{\sqrt{3}}{4} \right) = \frac{3e+1}{2} + \frac{(2e-1)\sqrt{3}}{4} =$$

$$= \frac{3e+1}{2} + \frac{e\sqrt{3} - \sqrt{3}}{4} =$$

$$= \frac{3+\sqrt{3}}{2} e + \frac{2-\sqrt{3}}{4}$$

(h) tečná rovina k ploše $z+1 = xe^y \cos z$
v bodě $(1,0,0)$

$$\Phi(x,y,z) = xe^y \cos z - z - 1$$

$$\nabla \Phi(x,y,z) = (e^y \cos z, xe^y \cos z, -xe^y \sin z - 1)$$

$$\frac{\partial \Phi}{\partial x} = e^y \cos z$$

$$\frac{\partial \Phi}{\partial y} = xe^y \cos z$$

$$\frac{\partial \Phi}{\partial z} = -xe^y \sin z - 1$$

$$\vec{n} = \nabla \Phi(1,0,0) = (1, 1, -1)$$

$$x + y - z + d = 0$$

$$1 + d = 0$$

$$d = -1$$

všechny roviny:

$$\underline{\underline{x + y - z - 1 = 0}}$$

$$(c) \min \text{ a max } f(x, y) = 6 - 4x - 3y$$

$$\text{na množině } M: x^2 + y^2 \leq 4y - 2x$$

$$x^2 - 2x + y^2 - 4y \leq 0$$

$$(x+1)^2 + (y-2)^2 \leq 5$$

$+1-1 \quad +4-4$

Levň je omezená uzavřená množina, což platí na celé množině:
vnitřek

$$M^\circ: (x+1)^2 + (y-2)^2 < 5$$

vnitřní podmínky extrémů, levnice musí být udána v dějství

$$\nabla f(x, y) = (-4, -3) \neq (0, 0)$$

kvůli množině

$$\partial M: (x+1)^2 + (y-2)^2 - 5 = g(x, y)$$

$$\nabla f(x, y) = \lambda \nabla g(x, y) \quad \text{podmínka } \nabla g(x, y) \neq (0, 0)$$

$$(-4, -3) = \lambda (2x+2, 2y-4)$$

$$-4 = 2\lambda(x+1)$$

$$-3 = 2\lambda(y-2)$$

$$(x+1)^2 + (y-2)^2 = 5$$

$$4/3 = \frac{x+1}{y-2}$$

$$y-2 = \frac{3}{4}(x+1)$$

$$(x+1)^2 + 9/16(x+1)^2 = 5$$

$$x^2 + 2x + 1 + 9/16(x^2 + 2x + 1) - 5 = 0 \quad | \cdot 16$$

$$\cancel{16x^2} + \cancel{32x} + 16 + \cancel{9x^2} + \cancel{18x} + 9 - \cancel{80} = 0$$

$$25x^2 + 50x - 55 = 0$$

$$y = 3/4(x+1) + 2 \quad x+1 = 4/5$$

$$5x^2 + 10x - 11 = 0$$

$$y = 3/4 \cdot 4/5 + 2 = 3/5 + 2$$

$$D = 100 + 44 \cdot 5 = 320$$

$$x_{1,2} = \frac{-10 \pm \sqrt{320}}{10} = \frac{-10 \pm 8\sqrt{5}}{10}$$

$$\left\{ \begin{array}{l} \frac{4}{\sqrt{5}} - 1, \quad y_1 = \frac{3}{\sqrt{5}} + 2 \\ -\frac{4}{\sqrt{5}} - 1, \quad y_2 = -\frac{3}{\sqrt{5}} + 2 \end{array} \right.$$

podzvěřené body: $(\frac{4}{\sqrt{5}} - 1, \frac{3}{\sqrt{5}} + 2)$

$(-\frac{4}{\sqrt{5}} - 1, -\frac{3}{\sqrt{5}} + 2)$

funkce f je $f(x_2, y_2) > f(x_1, y_1)$

podle věty o existenci maxima a minima je max v (x_2, y_2) a

min v (x_1, y_1)

na množině M

(d) l.h. min, l.h. max

$$f(x, y) = xy(3 - x + y) = 3xy - x^2y + xy^2$$

na otevrené množině $M: x > 0$

vnitřní podmínky \rightarrow vnější první derivace

$$\nabla f(x, y) = (y^2 - 2xy + 3, 2xy - x^2 + 3x) = (0, 0)$$

$$y^2 - 2xy + 3 = 0$$

$$2xy - x^2 + 3x = 0$$

$$x(2y - x + 3) = 0 \xrightarrow{x > 0} 2y - x + 3 = 0$$

$x = 2y + 3 = 3$

$$y(y - 2x + 3) = \begin{cases} y = 0, x = 3 \\ y - 2x + 3 = 0 \end{cases}$$

\Downarrow

Potom Hessova matice:

$$H_e(x, y) = \begin{pmatrix} -2y & 3 - 2x + 2y \\ 3 - 2x + 2y & 2x \end{pmatrix}$$
$$\begin{array}{l} 2x - y = 3 \\ x - 2y = 3 \quad | \cdot -2 \\ \hline 0 \quad 3y = -3 \\ y = -1 \end{array}$$

potom pro bod $(3, 0)$

$$H_e(3, 0) = \begin{pmatrix} 0 & -3 \\ -3 & 6 \end{pmatrix}$$

$$\begin{array}{l} 2x + 1 = 3 \\ 2x = 2 \quad x = 1 \end{array} \quad (x, y) = (1, -1)$$

$\Delta_1 = 0$
 $\Delta_2 = -9 < 0$
Sylvesterovo kritérium \Rightarrow sedlo v $(3, 0)$

$$H_e(1, -1) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \quad \begin{array}{l} \Delta_1 > 0 \\ \Delta_2 > 0 \end{array}$$

je pozitivně definitní, podle Sylv. krit.
v bodě $(1, -1)$ je tedy maximum

Najděte min a max fce $f(x,y) = 2x - y + 1$

na množině $M: x^2 + 2x + y^2 = 0 \quad (x+1)^2 + y^2 = 1$
 $(x+1)^2 - 1 + y^2 = 0$

Řešíme metodou Lagrangeových multiplikátorů.

$$g(x,y) = (x+1)^2 + y^2 - 1$$

potom, pokud $g(x,y) \neq 0$ platí, že:

$$\nabla f(x,y) = \lambda \nabla g(x,y)$$

$$\frac{\partial f}{\partial x} = 2 \quad \frac{\partial g}{\partial x} = 2x+2$$

$$\frac{\partial f}{\partial y} = -1 \quad \frac{\partial g}{\partial y} = 2y$$

$$(2, -1) = \lambda (2x+2, 2y)$$

$$\left. \begin{array}{l} 2 = 2\lambda(x+1) \\ -1 = 2\lambda y \end{array} \right\} \Rightarrow -2 = \frac{x+1}{y} \quad x+1 = -2y$$

$$(x+1)^2 + y^2 = 1$$

$$(-2y)^2 + y^2 = 1$$

$$4y^2 + y^2 = 1$$

$$5y^2 = 1$$

$$y^2 = \frac{1}{5} \Rightarrow y = \pm \frac{1}{\sqrt{5}}$$

$$x = -2y - 1$$

$$x = \frac{2\sqrt{5}}{5} - 1$$

$$x = \frac{-2\sqrt{5}}{5} - 1$$

dua body : $\left(-\frac{2\sqrt{5}}{5} - 1, \frac{\sqrt{5}}{5}\right) = a_1$
podle zřejmě
ž extrému

$$\left(\frac{2\sqrt{5}}{5} - 1, -\frac{\sqrt{5}}{5}\right) = a_2$$

podle věty o krajních extrému souvratně (min a max).

— M je uzavřená a omezená

$$f(x, y) = 2x - y + 1$$

$$\text{první bod: } \frac{-4\sqrt{5}}{5} - 2 - \frac{\sqrt{5}}{5} + 1 = -\sqrt{5} - 1$$

$$\frac{4\sqrt{5}}{5} - 2 + \frac{\sqrt{5}}{5} + 1 = \sqrt{5} - 1$$

vyhází, že bod a_2 je maximum a bod a_1 je minimum

- (i) rovnice roviny $\rightarrow \nabla f \dots H_e(\dots)$
 - (ii) lok. min max $\rightarrow \nabla f \dots$
 - (iii) směr největšího ústupu $\rightarrow \nabla f \dots$
 - (iv) extrémy fce s omezením \rightarrow Lagr.
 - (v) limity
- $\Delta \lambda_j = -1$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x+y} = \lim_{x \rightarrow 0} \frac{hx^2}{x+hx} = \lim_{x \rightarrow 0} \frac{x(hx)}{x(1+h)} =$$

$$y = kx$$

$$D_f = \mathbb{R} \setminus (0,0)$$

$$\lim_{x \rightarrow 0} \frac{kx}{1+h} = \frac{0}{1+h} = 0$$

pro $x = 0$

$$f(0,y) = \frac{0}{0+y} = \frac{y \rightarrow 0}{\rightarrow} 0$$

pro $y \neq 0$

(1.) $f(x,y) = x \sin(x+y^2) + 2$

určete v bode $a = (-1, 1)$

sečnou rovinnu a derivaci ve směru $\vec{n} = (\frac{\sqrt{2}}{2}, -\frac{1}{2})$

$$F(x,y,z) = x \sin(x+y^2) + 2 - z$$

f vrstevnice F

$$\frac{\partial F}{\partial x} = x \cos(x+y^2) + \sin(x+y^2)$$

$$\frac{\partial F}{\partial y} = x \cos(x+y^2) 2y$$

$$\frac{\partial F}{\partial z} = -1$$

$$\nabla F(x,y,z) = \begin{pmatrix} x \cos(x+y^2) + \sin(x+y^2) \\ x \cos(x+y^2) 2y \\ -1 \end{pmatrix}$$

$$a_z = -\sin(-1+1) + 2 = 2$$

$$a = (-1, 1, 2) \text{ grad } f(-1, 1) = (-1, -2)$$

$$\text{pokem } \vec{n} = \text{grad } F(-1, 1, 2) = (-1, -2, -1)$$

$$-x - 2y - z + d = 0$$

$$1 - 2 - 2 + d = 0$$

$$d = 3$$

$$\underline{\underline{-x - 2y - z + 3 = 0}}$$

$$\begin{aligned} \text{der ve směru } \vec{u} &\Rightarrow \nabla f(-1, 1) \cdot \vec{u} = \\ &= (-1, -2) \left(\frac{\sqrt{3}}{2}, -\frac{1}{2} \right) = \frac{-\sqrt{3}}{2} + 1 \end{aligned}$$

(2.) Loh. extrémů fce

$$f(x, y) = x^3 - 4x^2 + 2xy - y^2 \quad \text{Def} = \mathbb{R}$$

nulová podmínka existence: $\text{grad}(x, y) = (0, 0)$

$$\frac{\partial f}{\partial x} = 3x^2 - 8x + 2y = 0 \quad \frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$$

$$\frac{\partial f}{\partial y} = 2x - 2y = 0 \Rightarrow 2y = 2x$$

$$3x^2 - 6x = 0 \Rightarrow x(3x - 6) = 0 \quad \begin{cases} x=0, y=0 \\ x=2, y=2 \end{cases}$$

máme 2 body podle zřetel z extrému $(0,0), (2,2)$

Hessova matice:

$$H_f(x,y) = \begin{pmatrix} 6x-8 & 2 \\ 2 & -2 \end{pmatrix}$$

pro $(0,0)$

$$H_f(0,0) = \begin{pmatrix} \boxed{-8} & 2 \\ 2 & -2 \end{pmatrix}$$

$$\Delta_1 = -8 < 0$$

matice je neg.
definitní (Sylvester)

$$\Delta_2 = 16 - 4 > 0$$

lok. max.

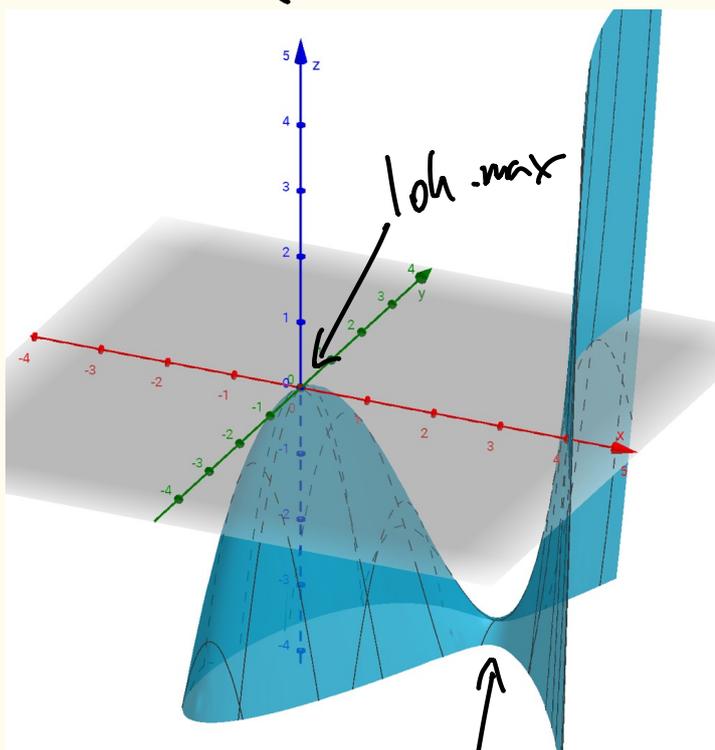
$$H_f(2,2) = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}$$

$$\Delta_1 = -2 < 0$$

indefinitní

$$\Delta_2 = 4 - 4 = 0$$

sedlový bod



sedlo

Příklad 5.3. Kruhový talíř o rovnici $x^2 + y^2 \leq 1$ je zahřátý na teplotu $T(x, y) = x^2 + 2y^2 - x$. Najděte nejteplejší a nejstudenější bod na talíři.

hledáme min a max fce $T(x, y) = x^2 + 2y^2 - x$
omezené na množině $M: x^2 + y^2 \leq 1$

rozdělime na

$$M^o: x^2 + y^2 < 1$$

vnitřní podmínka existence extrémů grad $T(x, y) = (0, 0)$

$$\text{grad } T(x, y) = (2x - 1, 4y) \quad \begin{matrix} 2x = 1 \\ 4y = 0 \end{matrix} \quad \begin{matrix} x = 1/2 \\ y = 0 \end{matrix}$$

$$\left(\frac{1}{2}\right)^2 + 0 < 1$$

$$\frac{1}{4} < 1 \quad \checkmark \quad \left(\frac{1}{2}, 0\right)$$

je podzvěřivý z rovnomu $x^2 + y^2 = 1$

$$\partial M = g(x, y) = x^2 + y^2 - 1$$

$$\nabla f(x, y) = \lambda \nabla g(x, y), \quad g(x, y) \neq 0$$

$$(2x - 1, 4y) = \lambda (2x, 2y)$$

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$$2x - 1 = 2\lambda x$$

$$4y = 2\lambda y$$

$$2x - 1 = 4x$$

$$x = -1/2$$

$$4 = 2\lambda$$

$$\lambda = 2$$

$$\underbrace{x^2 + y^2 = 1}$$

$$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\frac{1}{4} + y^2 = 1$$

$$y^2 = \frac{3}{4}$$

$$y = \pm \frac{\sqrt{3}}{2}$$

$$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

M je uzavřená omezená \rightarrow věta o existenci min. a max

bodů: $a_0 = (1/2, 0)$ $a_1 = (-1/2, \frac{\sqrt{3}}{2})$ $a_2 = (-1/2, -\frac{\sqrt{3}}{2})$

$$T(x,y) = x^2 + 2y^2 - x$$

$$T(a_0) = 1/4 - 1/2 = -1/4 \quad \left. \vphantom{T(a_0)} \right\} \text{minimum}$$

$$T(a_1) = 1/4 + \frac{3}{2} + \frac{1}{2} = \frac{9}{4} \quad \left. \vphantom{T(a_1)} \right\} \text{maxima}$$

$$T(a_2) = 1/4 + \frac{3}{2} + \frac{1}{2} = \frac{9}{4}$$

(i) $f(x,y,z) = xyz^2z^3$, $a_0 = (1, 2, 1)$ $\xrightarrow{(x,y,z)}$

$$T_2(a_0 + \vec{h}) = f(a_0) + f'(a_0)[a - a_0] + \frac{1}{2!} f''(a_0)[a - a_0]^2$$

$$f'(x,y,z) = \begin{pmatrix} y^2 z^3 & x 2y z^3 & x y^2 3z^2 \end{pmatrix} \rightarrow \begin{pmatrix} x-1 \\ y-2 \\ z-1 \end{pmatrix} = \vec{h}$$

$$f'(a_0) = (4, 4, 12)$$

$$f''(x,y,z) = \begin{pmatrix} 0 & 2yz^3 & y^3 3z^2 \\ 2yz^3 & 2xz^3 & 6xy z^2 \\ y^3 3z^2 & 6xy z^2 & 6x y^2 z \end{pmatrix} \Big|_{a_0} = \begin{pmatrix} 0 & 4 & 12 \\ 4 & 2 & 12 \\ 12 & 12 & 24 \end{pmatrix}$$

$$T_2(x,y,z) = 4 + (4, 4, 12) \begin{pmatrix} x-1 \\ y-2 \\ z-1 \end{pmatrix} + \frac{1}{2} (\vec{h}) \begin{pmatrix} \swarrow \\ \searrow \end{pmatrix} (\vec{h})$$

$$4 + 4x - 4 + 4y - 8 + 12z - 12 +$$

Taylor řádu 2 $f: \mathbb{R}^2 \rightarrow \mathbb{R}$

$$f(x, y) = e^{2x - y^2}$$

(5) Úhel mezi grafy: (má vyjít $\approx 50^\circ$)

$$z = \sqrt{x^2 - y^2} \quad z = x^3 + y^3 - 3xy \quad A = (1, 0, ?)$$

(6) zjistiťe hodnotu parametru s , aby se plochy γ (má vyjít $s = -2$)

$$x^2 + y^2 + (z - s)^2 = 18$$

$$z = x^2 + y^2 \quad \text{prohřaly pod úhlem } \frac{\pi}{2}$$

$$z^2 = x^2 - y^2 \rightarrow z = \sqrt{1} = 1$$

$$z = 1 + 0 - 0 = 1$$

$$f(x, y, z) = x^2 - y^2 - z^2$$

$$g(x, y, z) = x^3 + y^3 - 3xy - z$$

$$\nabla f(x, y, z) = (2x, -2y, -2z)$$

$$\nabla g(x, y, z) = (3x^2 - 3y, 3y^2 - 3x, -1)$$

$$\vec{m}_1 = \nabla f(1, 0, 1) = (2, 0, -2)$$

$$\vec{m}_2 = \nabla g(1, 0, 1) = (3, -3, -1)$$

$$\cos \varphi = \frac{\vec{m}_1 \cdot \vec{m}_2}{\|\vec{m}_1\| \cdot \|\vec{m}_2\|} = \frac{6 + 0 + 2}{\sqrt{4+4} \cdot \sqrt{9+9+1}} = \frac{8}{\sqrt{27}} = \frac{8}{3\sqrt{3}}$$

$$zx + 2y^2 - 2 = 0$$

$$f(x, y, z) = zx + 2y^2 - 2$$

$$\nabla f(x, y, z) = (z, 4y, x)$$

$$\nabla f(1, 0, 2) = (2, 0, 1)$$

$$\vec{n} = (2, 0, 1)$$

$$\cos \alpha = \frac{\vec{n} \cdot \vec{v}}{\|\vec{n}\| \cdot \|\vec{v}\|} = \frac{7}{\sqrt{5} \cdot \sqrt{17}} = \frac{7}{\sqrt{85}}$$

$$\alpha = \arccos\left(\frac{7}{\sqrt{85}}\right) \approx 41^\circ$$

$$z = 2x^2 + y^2 \quad A = (1, 0, 2)$$

$$g(x, y, z) = 2x^2 + y^2 - z$$

$$\nabla g(x, y, z) = (4x, 2y, -1)$$

$$\nabla g(1, 0, 2) = (4, 0, -1)$$

$$\vec{n} = (4, 0, -1)$$

(7.) tečná rovina k S , která je rovnoběžná se zadanou rovinou ρ

(a) plocha $S: x^2 + 4y^2 - z^2 = 4 = F(x, y, z)$

rovina $\rho: 2x + 2y + z = 0$

Hledáme všechny (x, y, z) takové, že $\nabla F(x, y, z) = \alpha (2, 2, 1)$

a platí, že $F(x, y, z) = 4$

(*) $\alpha^2 = 4$

$\alpha = \pm 2$

$$\nabla F(x, y, z) = (2x, 8y, -2z)$$

$$2x = 2\alpha \quad x = \alpha$$

$$8y = 2\alpha \quad 4y = \alpha \Rightarrow y = \frac{\alpha}{4}$$

$$-2z = \alpha \quad z = -\frac{\alpha}{2}$$

$$x^2 + 4y^2 - z^2 = 4 \Rightarrow \alpha^2 + \frac{\alpha^2}{4} - \frac{\alpha^2}{4} = 4 \quad (*)$$

potom dostáváme dva body: $(2, \frac{1}{2}, -1)$ pro $d=2$

$(-2, -\frac{1}{2}, 1)$ pro $d=-1$

potom rovnice roviny bude kde $(x_0, y_0, z_0) = (2, \frac{1}{2}, -1)$

$$2x_0(x-x_0) + 8y_0(y-y_0) - 2z_0(z-z_0) = 0 \quad \leftarrow$$

$$2 \cdot 2(x-2) + 4 \cdot (y - \frac{1}{2}) + 2(z+1) = 0$$

$$4x - 8 + 4y - 2 + 2z + 2 = 0$$

$$4x + 4y + 2z - 8 = 0$$

$$\underline{\underline{2x + 2y + z - 4 = 0}} \quad \text{kde } (x_0, y_0, z_0) = (-2, -\frac{1}{2}, 1)$$

nebo $2x_0(x-x_0) + 8y_0(y-y_0) - 2z_0(z-z_0) = 0 \quad \leftarrow$

$$-4(x+2) - 4(y + \frac{1}{2}) - 2(z-1) = 0$$

$$-4x - 8 - 4y - 2 - 2z + 2 = 0$$

$$-4x - 4y - 2z - 8 = 0$$

$$\underline{\underline{2x + 2y + z + 4 = 0}}$$

$$(8) (a) f: xyz = 1$$

$$A = (1, 1, 1)$$

$$g: x^2 + 2y^2 + 3z^2 = 6$$

$$\nabla f(x, y, z) = (yz, xz, xy)|_A = (1, 1, 1) = \vec{u}$$

$$\nabla g(x, y, z) = (2x, 4y, 6z)|_A = (2, 4, 6) = \vec{v}$$

$$\vec{w} = \vec{u} \times \vec{v} = (6 - 4, 2 - 6, 4 - 2) = (2, -4, 2)$$

$$\underline{\underline{(2, -4, 2)t + A, t \in \mathbb{R}}}$$

$$(9) S: x^2 + 2y^2 + 3z^2 = 6$$

je rovnoběžná s S :

$$S: x - 2y + 3z = 0$$

$$(2x, 4y, 6z) = d(1, -2, 3)$$

$$F(x, y, z) = x^2 + 2y^2 + 3z^2$$

$$2x = d \Rightarrow x = \frac{d}{2}$$

$$2y = -d \Rightarrow y = -\frac{d}{2}$$

$$2z = d \Rightarrow z = \frac{d}{2}$$

$$\nabla F(x, y, z) = (2x, 4y, 6z)$$

dva body:

$$(1, -1, 1)$$

$$x^2 + 2y^2 + 3z^2 = 6$$

$$\frac{d^2}{4} + 2 \frac{d^2}{4} + 3 \frac{d^2}{4} = 6$$

$$\frac{6d^2}{4} = 6$$

$$6d^2 = 6 \cdot 4$$

$$d^2 = 4 \quad d = \pm 2$$

$$2x_0(x-x_0) + 4y_0(y-y_0) + 6z_0(z-z_0) = 0$$

$$2(x-1) - 4(y+1) + 6(z-1) = 0$$

$$2x - 2 - 4y - 4 + 6z - 6 = 0$$

$$2x - 4y + 6z - 12 = 0 \Rightarrow \underline{\underline{x - 2y + 3z - 6 = 0}}$$

$$(2) f(x, y) = y^2 - x^2$$

$$M: x^2 + 4y^2 = 4 \quad g(x, y) = x^2 + 4y^2 - 4$$

$$\nabla f(x, y) = (-2x, 2y)$$

$$\nabla g(x, y) = (2x, 8y) \quad \text{potem } \nabla g(x, y) \neq 0$$

$$L = y^2 - x^2 - \lambda(x^2 + 4y^2 - 4) = y^2 - x^2 - \lambda x^2 - 4\lambda y^2 + 4\lambda$$

$$\frac{\partial L}{\partial x} = 0 \quad \frac{\partial L}{\partial y} = 0 \quad \frac{\partial L}{\partial \lambda} = 0$$

$$\rightarrow x=0 \vee \lambda = -1$$

$$-2x - 2\lambda x = 0 \quad -2x(1 + \lambda) = 0$$

$$2y - 8\lambda y = 0 \quad 2y(1 - 4\lambda) = 0$$

$$\rightarrow y=0 \vee \lambda = 1/4$$

$$-x^2 - 4y^2 + 4 = 0$$

$$y^2 = 1 \text{ przy } x=0$$

$$y = \pm 1$$

extrémum w $(0, \pm 1)$?

$$y=0 \rightarrow x = \pm 2 \quad (\pm 2, 0)$$

max

min

$$y^2 - x^2$$

$$(b) f(x, y) = x^2 + y^2 \quad M = \{(x, y) \in \mathbb{R}^2 \mid xy = 1\}$$

$$\text{warunkowa podmiernka } g(x, y) = 0, \quad g(x, y) = xy - 1$$

$$\text{potem } L(x, y) = x^2 + y^2 + \lambda(xy - 1)$$

$$\frac{\partial L}{\partial x} = 2x + \lambda y = 0 \quad \frac{\partial L}{\partial y} = 2y + \lambda x = 0 \quad \frac{\partial L}{\partial \lambda} = xy - 1 = 0$$

$$2x + \lambda y = 0 \quad 2x = -\lambda y \quad \frac{x}{y} = -\frac{\lambda}{x}$$

$$2y + \lambda x = 0 \quad 2y = -\lambda x \quad \frac{y}{x} = -\frac{\lambda}{y}$$

$$xy - 1 = 0 \quad x^2 = y^2$$

$$xy = 1$$

$$x^2 y^2 = 1$$

$$y^4 = 1 \quad y = \pm 1 \quad x = \pm 1$$

dua body $(1, 1)$ $(-1, -1)$ pada titik tersebut

f shows non-convex on $M \rightarrow$ need max \Rightarrow minima

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sqrt{kx^2+1} - 1}{x+y}$$

$$y = kx \quad \frac{\sqrt{kx^2+1} - 1}{x+kx} = \frac{\sqrt{kx^2+1} - 1}{x(k+1)} \cdot \frac{\sqrt{kx^2+1} + 1}{\sqrt{kx^2+1} + 1} =$$

$$= \frac{kx^2 + 1 - 1}{x(k+1)(\sqrt{kx^2+1} + 1)} = \frac{kx}{(k+1)(\sqrt{kx^2+1} + 1)} \xrightarrow{x \rightarrow 0} 0$$

$$f(x, y) = 2x - y + 1$$

$$g(x, y) = 0 \rightarrow g(x, y) = x^2 + 2x + y^2 = (x+1)^2 + y^2 - 1$$

$$h(x, y) = 2x - y + 1 - \lambda(x^2 + 2x + y^2)$$

$$\frac{\partial h}{\partial x} = 2 - 2\lambda x - 2\lambda = 0$$

$$2\lambda x + 2\lambda = 2$$

$$2\lambda(x+1) = 2$$

$$\frac{x+1}{y} = -2$$

$$\frac{\partial h}{\partial y} = -1 - 2\lambda y = 0$$

$$2\lambda y = -1$$

$$x+1 = -2y$$

$$\frac{\partial h}{\partial \lambda} = (x+1)^2 + y^2 - 1 = 0$$

$$(x+1)^2 + y^2 - 1 = 0$$

$$(2y)^2 + y^2 = 1$$

$$5y^2 = 1$$

$$y = \pm \frac{\sqrt{5}}{5}$$

dua body

$$\left(\frac{2\sqrt{5}}{5} - 1, \frac{\sqrt{5}}{5}\right)$$

$$x = 2y - 1$$

$$x = \pm \frac{2\sqrt{5}}{5} - 1$$

$$\left(-\frac{2\sqrt{5}}{5} - 1, \frac{\sqrt{5}}{5}\right)$$

M je uzavřená omezená \rightarrow f na ní nabývá svého min max dle věty
o každé funkci extrémní

tečné rovny ke kvadratickým zadáním jako plynů desek
ploch v předepsaném bodě

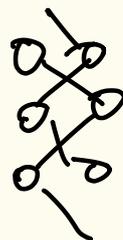
$$xyz - 1 = 0 = f(x, y, z)$$

$$A = (1, 1, 1)$$

$$x^2 + 2y^2 + 3z^2 - 6 = 0 = g(x, y, z)$$

$$\nabla f(x, y, z) = (yz, xz, xy)|_A = (1, 1, 1) = \vec{u}$$

$$\nabla g(x, y, z) = (2x, 4y, 6z)|_A = (2, 4, 6) = \vec{v}$$



hledám normálový vektor plochy tečné, tím hledám vektor kolmý na
 \vec{u} a \vec{v} zároveň

$$\vec{w} = \vec{u} \times \vec{v} = (6 - 4, 2 - 6, 4 - 2) = (2, -4, 2)$$

potom plocha: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \text{span}\left(\begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix}\right)$

$$\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} - 1 = F(x, y, z)$$

$$g(x, y, z) = 4x + 2y + z - 3$$

$$\nabla F(x, y, z) = \left(\frac{2x}{25}, \frac{y}{8}, \frac{2z}{9} \right) = \alpha (4, 2, 1)$$

$$\frac{2x}{25} = 4\alpha \quad 2x = 100\alpha \quad x = 50\alpha$$

$$\frac{y}{8} = 2\alpha \quad y = 16\alpha$$

$$\frac{2z}{9} = \alpha \quad 2z = 9\alpha \quad z = \frac{9}{2}\alpha$$

$$\frac{(50\alpha)^2}{25} + \frac{(16\alpha)^2}{16} + \frac{\left(\frac{9}{2}\alpha\right)^2}{9} = 1$$

$$100\alpha^2 + 16\alpha^2 + \frac{9}{4}\alpha^2 = 1 \quad | \cdot 4$$

$$400\alpha^2 + 64\alpha^2 + 9\alpha^2 = 4$$

$$473\alpha^2 = 4$$

$$\alpha = \pm \frac{2}{\sqrt{473}}$$

$$x = \frac{100}{\sqrt{473}} \quad y = \frac{32}{\sqrt{473}} \quad z = \frac{9}{\sqrt{473}}$$

$$4x + 2y + z \pm \sqrt{473} = 0$$

$$ax + by + cz + d = 0$$

$$4x + 2y + z + d = 0$$

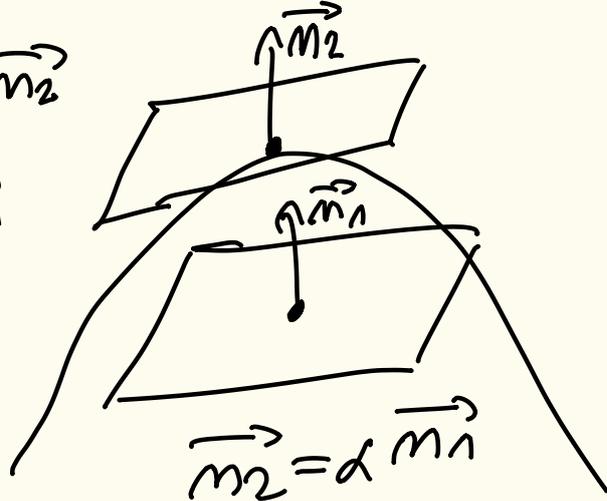
$$\frac{400}{\sqrt{473}} + \frac{64}{\sqrt{473}} + \frac{9}{\sqrt{473}} + d = 0 \quad \pm d = \frac{473}{\sqrt{473}} = \frac{\sqrt{473} \cdot \sqrt{473}}{\sqrt{473}} \Rightarrow d = \pm \sqrt{473}$$

$$F(x, y, z) = x^2 + 4y^2 - z^2 - 4 \vec{m}_2$$

$$G(x, y, z) = 2x + 2y + z \vec{m}_1$$

$$\nabla F(x, y, z) = (2x, 8y, -2z)$$

$$\nabla G(x, y, z) = (2, 2, 1)$$



$$x = d \quad \text{pro v\u011bj } d \neq 0$$

$$2x = 2d$$

$$8y = 2d$$

$$-2z = d \quad z = \frac{d}{4}$$

Hledáme (x, y, z) taková, že

$$\nabla F(x, y, z) = \lambda \nabla G(x, y, z) \quad \text{pro v\u011bj } \lambda \neq 0$$

$$z = \frac{d}{4}$$

$$x^2 + 4y^2 - z^2 = 4$$

$$d^2 + 4 \frac{d^2}{16} - \frac{d^2}{4} = 4$$

$$d^2 + \frac{d^2}{4} - \frac{d^2}{4} = 4$$

$$d = \pm 2$$

dua body $(2, \frac{1}{2}, -1) A_0$

$(-2, -\frac{1}{2}, 1) A_1$

potom tečná rovina:

$$2x + 2y + z + d = 0$$

pro A_0 :

$$4 + 1 - 1 = -d$$

$$d = 4$$

potom roviny

$$2x + 2y + z \pm 4 = 0$$

3.8] $f(x, y) = \sqrt{x^2 + y^2}$ $A = (-1, 0)$
 $\nabla f(x, y) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right)$ $\nabla f(-1, 0) = (-1, 0)$

$g(x, y) = e^{\sin(xy)}$

$\nabla g(x, y) = \left(e^{\sin xy} \cos xy, x e^{\sin xy} \cos xy \right)$

$\nabla g(-1, 0) = (0, -1)$

$\nabla F(x, y, z) = (-1, 0, -1)$

$f(x, y) = \sqrt{x^2 + y^2}$

$\nabla G(x, y, z) = (0, -1, -1)$

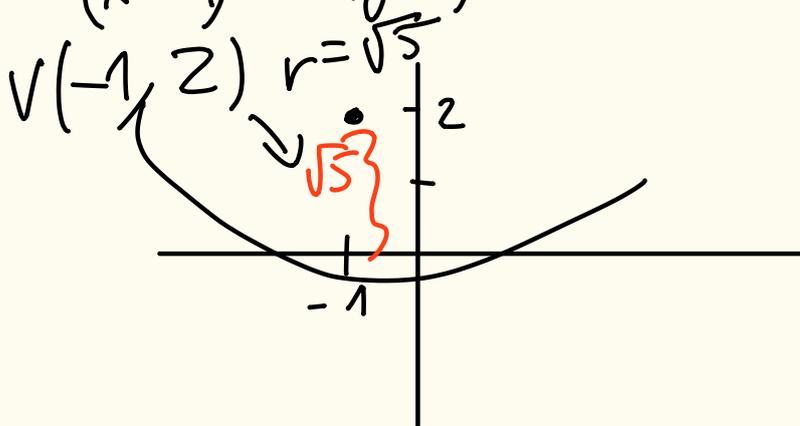
$F(x, y, z) = 0 \Rightarrow \sqrt{x^2 + y^2} = z$

$\cos \alpha = \frac{1 + 0 + 1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2} \Rightarrow \alpha = 60^\circ \left(= \frac{\pi}{3} \right)$

$M: x^2 + 2x + y^2 - 4y \leq 0$

$(x + 1)^2 - 1 + (y - 2)^2 - 4 \leq 0$

$(x + 1)^2 + (y - 2)^2 \leq (\sqrt{5})^2$



$$f(x, y) = 6 - 4x - 3y$$

$$M: (x+1)^2 + (y-2)^2 - 5 \leq 0$$

rozkládáme množinu na

$$M^o: g(x, y) = (x+1) + (y-2)^2 - 5 < 0$$

vnitřní podmínky: $\nabla f(x, y) = 0$ na M^o

$$\nabla f(x, y) = (-4, -3) = (0, 0) \rightarrow \text{nikdy}$$

$$\begin{aligned} -4 &= 0 && \text{nikdy} \\ -3 &= 0 && \text{nikdy} \end{aligned}$$

na M^o nejsou extrémny

$$\partial M: g(x, y) = \underbrace{(x+1)^2 + (y-2)^2 - 5}_{=0}$$

$$h(x, y) = f(x, y) - \lambda g(x, y) = 6 - 4x - 3y - \lambda((x+1)^2 + (y-2)^2 - 5)$$

$$\frac{\partial h}{\partial x} = -4 - 2\lambda x - 2\lambda = 0 \quad (1) \quad \begin{aligned} 2\lambda + 2\lambda x &= -4 \\ 2\lambda(1+x) &= -4 \end{aligned}$$

$$\frac{\partial h}{\partial y} = -3 - 2\lambda y + 4\lambda = 0 \quad (2) \quad \begin{aligned} 2\lambda y - 2\lambda &= -3 \\ 2\lambda(y-2) &= -3 \end{aligned}$$

$$\frac{\partial h}{\partial \lambda} = \underbrace{- (x+1)^2 - (y-2)^2 + 5}_{=0} = 0 \quad (3)$$

$$(x+1)^2 + (y-2)^2 = 5$$

$$(x+1)^2 + \left(\frac{3(1+x)}{4}\right)^2 = 5$$

$$\frac{1+x}{y-2} = \frac{4}{3}$$

$$3(1+x) = 4(y-2)$$

$$y-2 = \frac{3(1+x)}{4}$$

$$(x+1)^2 + \frac{9(x+1)^2}{16} = 5$$

$$(x+1)^2 \left(1 + \frac{9}{16}\right) = 5$$

$$(x+1)^2 \frac{25}{16} = 5 \quad | :5$$

$$(x+1)^2 \frac{5}{16} = 1$$

$$5(x+1)^2 = 16$$

$$(x+1)^2 = \frac{16}{5}$$

$$x+1 = \pm \frac{4\sqrt{5}}{5}$$

$$a_0 = \left(\frac{4\sqrt{5}}{5} - 1, \frac{3\sqrt{5}}{5} + 2 \right)$$

$$a_1 = \left(-\frac{4\sqrt{5}}{5} - 1, -\frac{3\sqrt{5}}{5} + 2 \right)$$

podle věty o nabývání maxima a minima (M je uzavřená množina)
se bude nabývat min a max

$$b - 4x - 3y$$

$$\begin{array}{ll} \text{v } a_0 & \text{min} \\ a_1 & \text{max} \end{array}$$

$$d_1 = \text{šel grad } f \text{ od } U = (1, 2, e^2) \quad A = (1, 1, e)$$

$$d_2 = \text{šel grad } f \text{ od } V = (2, 1, e^2/42)$$

$$\text{grad } f = (e+1, e)$$

$$f(x, y) = x^3 - y^3 - 2xy + 6$$

$$\nabla f(x, y) = (3x^2 - 2y, -3y^2 - 2x)$$

nutri podmínky existence extrému

$$\nabla f(x, y) = 0$$

$$\begin{aligned} 3x^2 - 2y = 0 &\rightarrow 2y = 3x^2 \\ -3y^2 - 2x = 0 &\quad y = \frac{3}{2}x^2 \end{aligned}$$

$$-3\left(\frac{3}{2}x^2\right)^2 - 2x = 0$$

$$-3 \frac{9}{4} x^4 - 2x = 0$$

$$-\frac{27}{4} x^4 - 2x = 0$$

$$\frac{1}{4} x (27x^3 + 8) = 0$$

$$x = 0 \quad 27x^3 = -8$$

$$x^3 = -\frac{8}{27} \Rightarrow x = \sqrt[3]{-\frac{8}{27}} = -\frac{2}{3}$$

$$y = \frac{3}{2} \left(-\frac{2}{3}\right)^2 = \frac{3}{2} \cdot \frac{4}{9} = \frac{12}{18} = \frac{6}{9} = \frac{2}{3}$$

podzřelý bod v $(-\frac{2}{3}, \frac{2}{3})$ podzřelý bod v $(0, 0)$

$$\text{Hessian } H_f(x, y) = \begin{pmatrix} 6x & -2 \\ -2 & -6y \end{pmatrix}$$

$$H_f\left(-\frac{2}{3}, \frac{2}{3}\right) = \begin{pmatrix} -4 & -2 \\ -2 & -4 \end{pmatrix} \begin{array}{l} \Delta_1 < 0 \text{ max} \\ \Delta_2 > 0 \end{array}$$

$$H_f(0, 0) = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} = \begin{array}{l} \Delta_1 \neq 0 \\ \Delta_2 > 0 \end{array} \text{ sedlo}$$

$$f(x, y) = xy(3 - x + y) = 3xy - x^2y + xy^2$$

na $M: x > 0$

$$\nabla f(x, y) = (3y - 2xy + y^2, 3x - x^2 + 2xy)$$

notas podmínky pro existenci extrémů:

$$\nabla f(x, y) = (0, 0)$$

$$3y - 2xy + y^2 = 0$$

$$3x - x^2 + 2xy = 0$$

$$y(3 - 2x + y) = 0 \quad y = 0 \vee 3 - 2x + y = 0$$

$$x(3 - x + 2y) = 0$$

$$x \neq 0 \vee 3 - x + 2y = 0$$

$x > 0$ dle M

$$(3, 0)$$

$$x = 3 + 2y$$

$$(1, -1)$$

$$3 - 2(3 + 2y) + y = 0$$

Hessián:

$$H_f(x, y) = \begin{pmatrix} -2y & 3 - 2x + 2y \\ 3 - 2x + 2y & 2x \end{pmatrix}$$

$$3 - 6 - 4y + y = 0$$

$$-3 = 3y$$

$$y = -1$$

$$H_f(3, 0) = \begin{pmatrix} 0 & \\ & \end{pmatrix} \rightarrow \text{sedlo (indefinitní)}$$

$$H_f(1, -1) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{matrix} \Delta_1 = 2 > 0 \\ \Delta_2 = 4 - 1 > 0 \end{matrix} \text{ lokální minimum} \\ \text{pos. definitní}$$

$$f(x,y) = \frac{1}{(x^2 + 2y^2 + 1)} \quad x=2 \quad y=1$$

$$A(2,1,?)$$

$$\nabla f(x,y) = \left(\frac{-2x}{(x^2 + 2y^2 + 1)^2}, \frac{-4y}{(x^2 + 2y^2 + 1)^2} \right)$$

$$\nabla f(2,1) = \left(\frac{-4}{(4 + 2 + 1)^2}, \frac{-4}{(4 + 2 + 1)^2} \right) =$$

$$= \left(\frac{-4}{7^2}, \frac{-4}{7^2} \right) \rightarrow \text{vektor nejv. stupňní}$$

$$\text{nejv. klesání} \rightarrow \vec{v} = (1, 1)$$

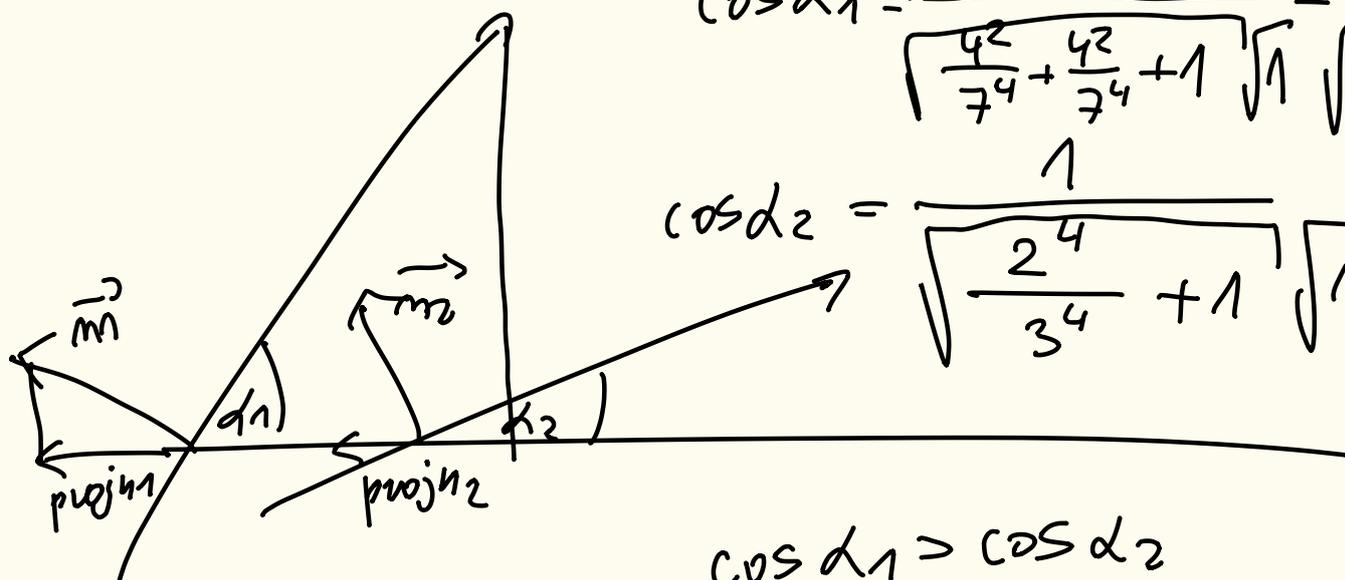
$$x=0 \quad y=1 \quad F(x,y,z) = f(x,y) - z$$

$$\vec{m}_A = \left(\frac{-4}{7^2}, \frac{-4}{7^2}, -1 \right)$$

$$\vec{m}_B = \left(0, \frac{-4}{3^2}, -1 \right) \quad \vec{z} = (0, 0, 1)$$

$$\cos \alpha_1 = \frac{1 - 1}{\sqrt{\frac{4^2}{7^4} + \frac{4^2}{7^4} + 1}} \sqrt{1} = \frac{1}{\sqrt{\frac{2^5}{7^4} + 1}}$$

$$\cos \alpha_2 = \frac{1}{\sqrt{\frac{2^4}{3^4} + 1}} \sqrt{1}$$



$$\cos \alpha_1 > \cos \alpha_2$$

$$\alpha_1 < \alpha_2$$

$$\left\| \left(-\frac{4}{7^2}, -\frac{4}{7^2} \right) \right\| =$$

$$= \sqrt{\frac{2^5}{7^4}}$$

$$\left\| \left(0, \frac{-4}{3^2} \right) \right\| =$$

$$= \sqrt{\frac{2^4}{3^4}}$$

B má větší spád

$$f(x, y, z) =$$

$$x^2 + y^2 + (z - 5)^2 - 18 = 0$$

$$g(x, y, z) = x^2 + y^2 - z = 0 \quad \begin{matrix} \nearrow z > 0 \\ \text{úhel je } \frac{\pi}{2} \end{matrix}$$

$$\nabla f(x, y, z) = (2x, 2y, 2z - 10)$$

$$\nabla g(x, y, z) = (2x, 2y, -1) \rightarrow \nabla g \neq 0$$

$$0 = \nabla f \cdot \nabla g$$

$$4x^2 + 4y^2 - 2z + 10 = 0$$

$$z = x^2 + y^2 \quad \curvearrowright$$

$$x^2 + y^2 + (z - 5)^2 = 18$$

$$4(x^2 + y^2) - 2z + 10 = 0$$

$$x^2 + y^2 + (z - 5)^2 = 18$$

$$z + (z - 5)^2 = 18$$

$$4z - 2z + s = 0$$

$$z + z^2 - 2zs + s^2 = 18$$

$$\rightarrow 2z + s = 0$$

$$s = -2z$$

$$z + z^2 - 2z(-2z) + 4z^2 = 18$$

$$z + z^2 + 4z^2 + 4z^2 = 18$$

$$9z^2 + z = 18$$

$$9z^2 + 9z - 18 = 0$$

$$D = \sqrt{81 + 648} = 27$$

$$z^2 + z - 2 = 0$$

$$\sqrt{D} = \sqrt{1 + 8} = 3$$

$$z_{1,2} = \frac{-1 \pm 3}{2} \quad \begin{matrix} \times \\ 1 \end{matrix}$$

$$\underline{\underline{s = -2z = -2}}$$