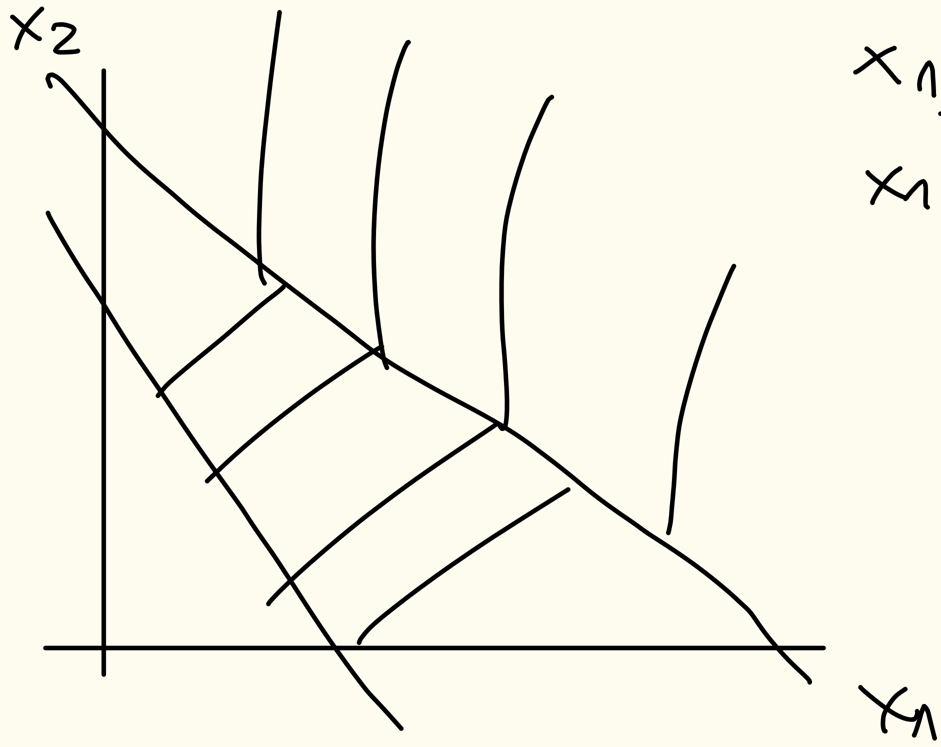


$x_1, x_2 \geq 0$

$x_1 + x_2 \geq 1 = b$

$2 = b + \Delta$

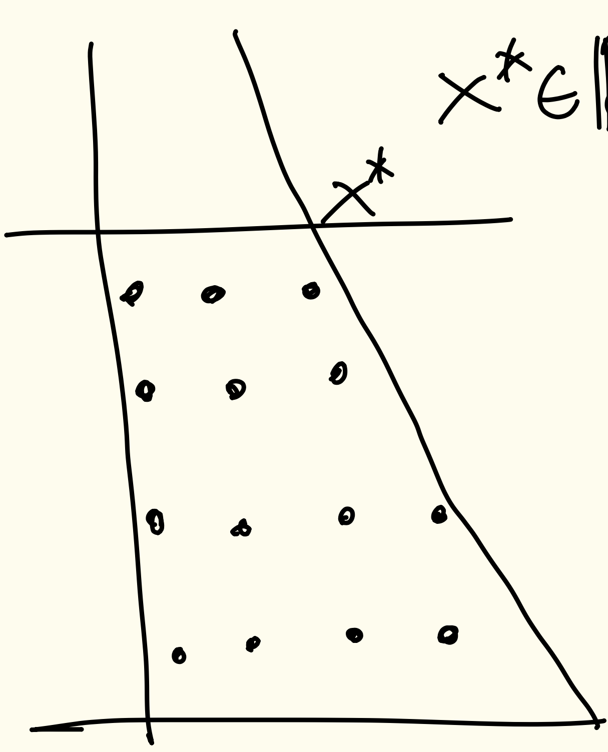
nadholt b:

$\max b^T y = b^T y^*$

$$
\begin{array}{lrcl}
\min & 2x_1 + 5x_2 + 6x_3 & = & \color{green}{5.402} \\
 & 2x_1 + x_2 + 2x_3 & \geq & 3.\color{red}{01} \\
 & x_1 + 2x_2 + 2x_3 & \geq & 1 \\
 & x_1 + 3x_2 + x_3 & \geq & 3 \\
 & -x_1 + x_2 - 2x_3 & \geq & -1 \\
 & x_1 & \geq & 0 \\
 & x_2 & \geq & 0 \\
 & x_3 & \geq & 0
\end{array}
\qquad
\begin{array}{rlrcl}
 & \max & 3.\color{red}{01}y_1 + y_2 + 3y_3 - y_4 & = & \color{green}{5.4}\color{red}{02} \\
\color{red}{0.2 =} & & y_1 & \geq & 0 \\
\color{red}{0 =} & & y_2 & \geq & 0 \\
\color{red}{1.6 =} & & y_3 & \geq & 0 \\
\color{red}{0 =} & & y_4 & \geq & 0 \\
 & & 2y_1 + y_2 + y_3 - y_4 & \leq & 2 \\
 & & y_1 + 2y_2 + 3y_3 + y_4 & \leq & 5 \\
 & & 2y_1 + 2y_2 + y_3 - 2y_4 & \leq & 6
\end{array}
$$

$$f(\mathbf{b}) + \boldsymbol{\Delta}^T \mathbf{y}^* = \color{green}{5.4} + (\color{red}{0.01}, 0, 0, 0)^T (0.2, 0, 1.6, 0) = \color{green}{5.402}$$

$b \in \mathbb{R}^4$ "$(3,1,3,-1)$ $+$ Δ

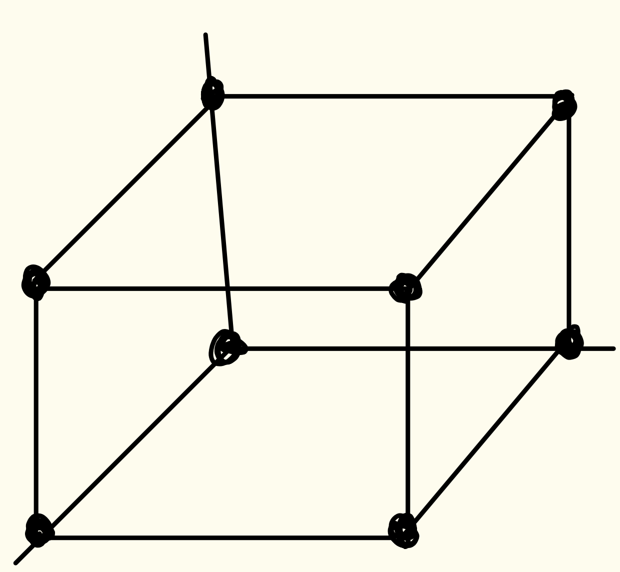


$x^* \in \mathbb{R} \setminus \mathbb{Q}$

$x_i \in \{0, 1\}$

$n = 3$

$x = (x_1, x_2, x_3)$



Přiřazovací problém

permutační matice

$$
\begin{array}{c|ccc}
 & 1' & 2' & 3' \\
\hline
1 & 0 & 1 & 0 \\
2 & 0 & 0 & 1 \\
3 & 1 & 0 & 0
\end{array}
$$

$x_{ij} \in \{0,1\}$ problém s bin proměnnými

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

za podmínek

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1 \dots n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1 \dots n$$

LP relaxace: $x_{ij} \in [0,1]$

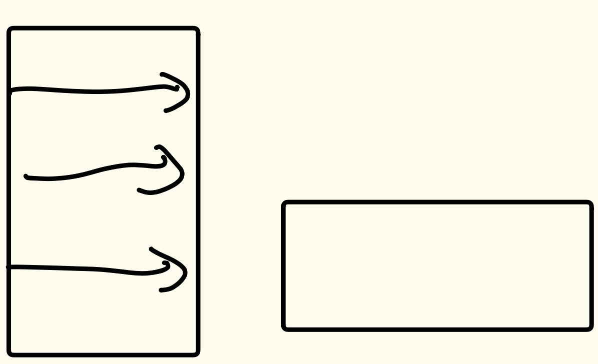
potom matice (už ne permutační) může být

$$\begin{bmatrix} 0{,}3 & 0{,}7 & 0 \\ 0{,}6 & 0{,}3 & 0{,}1 \\ 0{,}1 & 0 & 0{,}9 \end{bmatrix}$$

LP relaxace i původní úloha mají stejnou úlohu

mezi optimálními řešeními LP relaxace existuje celočíselné

$A \in \mathbb{R}^{n \times n}$



$\in \mathbb{R}^{n^2}$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} = \langle C, X \rangle$$

$$X \mathbb{1} = \mathbb{1}$$

$$\mathbb{1}^T X = \mathbb{1}^T$$

minimal vertex cover $x_i \in \{0,1\}, i \in V$

$$\min \sum_{i \in V} x_i$$

za podm. $x_i + x_j \geq 1$

$\min \; x_1 + x_2 + x_3$

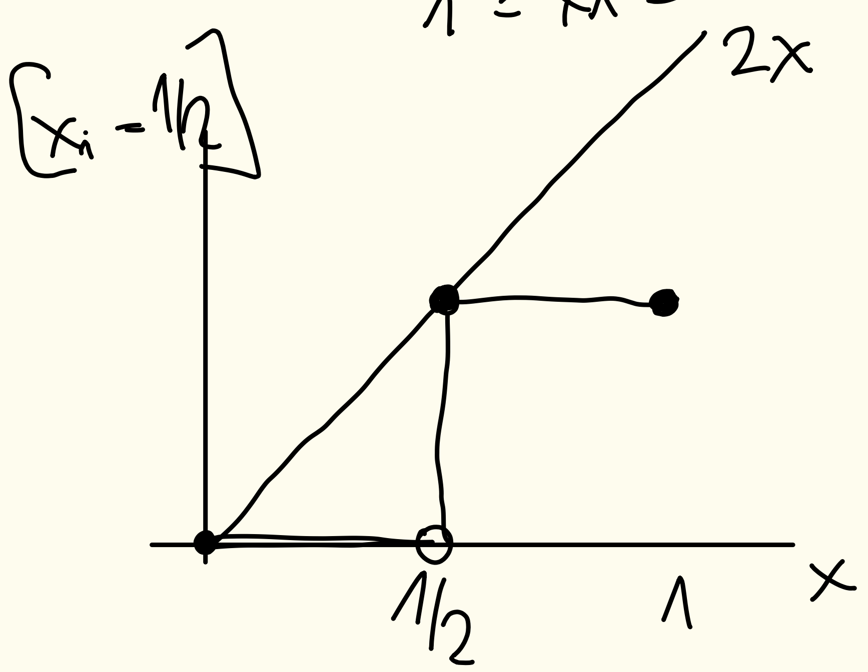
z.p. $\left.\begin{aligned} x_1 + x_2 &\geq 1 \\ x_1 + \quad x_3 &\geq 1 \\ x_2 + x_3 &\geq 1 \end{aligned}\right\} \oplus$

$1 \geq x_i \geq 0$

$2x_1 + 2x_2 + 2x_3 \; /:2$

$\vee\!|$

$3 \; /:2$



$x_i + x_j \geq 1$

$\Downarrow$

$x_i \geq 1/2$ nebo $x_j \geq 1/2$

$\Downarrow$

$\bar{x}_i = 1$ nebo $\bar{x}_j = 1$

$\Downarrow$

$\bar{x}_i + \bar{x}_j \geq 1$

$$\bar{y} = \sum_i \bar{x}_i \leq 2 \sum_i x_i$$

$$= 2y \leq 2y$$

1) Restrikce hodnot

$x \in \{a_1, \dots, a_k\} \subseteq \mathbb{R}$

2) kapitál k do 3 projektů

$x = (x_1, x_2, x_3) \in \{0,1\}$

$c \dots$ výnos, $\max c^T x$

z.p. $a^T x \leq k$

(a) alespoň 2 projekty

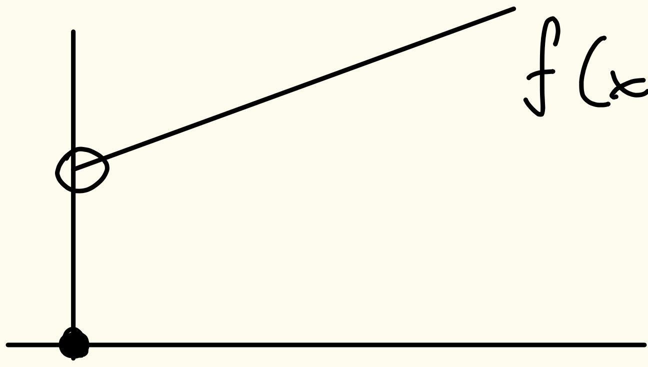
$x_1 + x_2 + x_3 \geq 2$

(b) Do 1 $\Leftrightarrow$ do 3

$x_1 = x_3$

(c) $2 \Rightarrow 1$

$x_2 \leq x_1$

3)  $f(x) = \begin{cases} 0 & x=0 \\ cx+d, & x>0 \end{cases}$ (d) 3 => 71 $x_3 \leq 1 - x_1$

$y \in \{0,1\}$

$(y = 1 \iff x > 0)$

$K^{-1} y \leq x \leq K y$

max $x_1 + x_2$

z.p. $2x_1 + x_2 \geq 6$

NEBO

$x_1 + 2x_2 \geq 7$

$x_1, x_2 \geq 0$

není konvex

$y \in \{0,1\}$

$2x_1 + x_2 \geq 6y$

$x_1 + 2x_2 \geq 7$