Úlohy s omezením ve tvaru rovnosti

hledáme extrémy f vázané rovnostmi

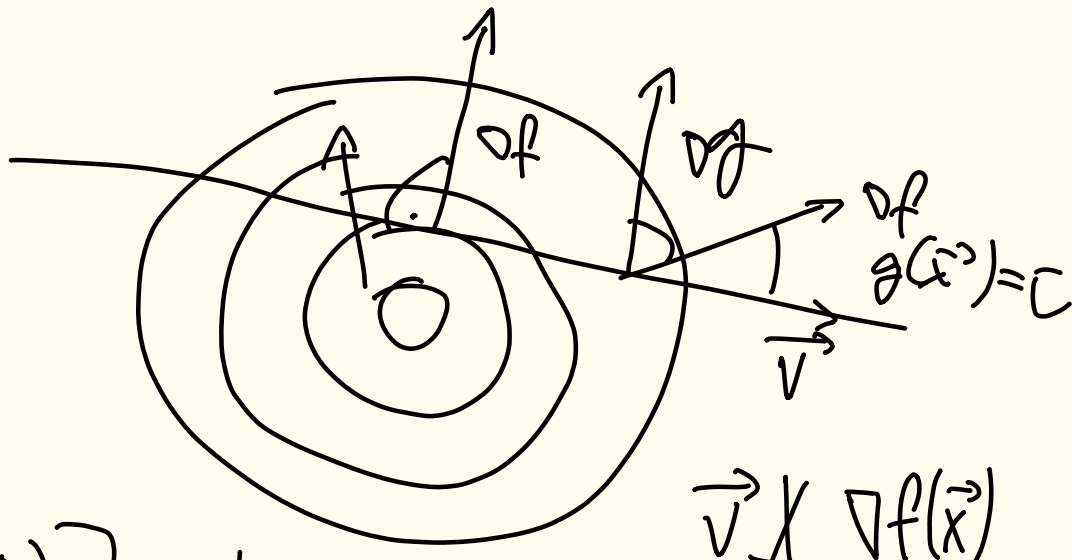
$$C \quad \vec{x} \quad d$$

$$\begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \\ c_{m1} & \cdots & c_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}$$

$$g'(x) = C$$

neboli

$$\nabla g(x) = C^T$$



$$\vec{v} \not\perp \nabla f(\vec{x})$$

$$g'(\vec{x}) = \begin{matrix} g_1 \to \\ \\ g_m \to \end{matrix} \begin{bmatrix} D_1 g_1(\vec{x}) & \cdots & D_n g_1(\vec{x}) \\ \vdots & & \\ D_1 g_m(\vec{x}) & \cdots & D_n g_m(\vec{x}) \end{bmatrix} =$$

$$\vec{g}(\vec{x}) = C\vec{x} - d =$$

$$= \begin{bmatrix} c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n - d_1 \\ \\ c_{m1}x_1 + \cdots \qquad c_{mn}x_n - d_m \end{bmatrix}$$

$$= \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ & & & \\ c_{m1} & \cdots & & c_{mn} \end{bmatrix} = C$$

Předp: $\exists \vec{v} \in \mathbb{R}^n : \underbrace{\vec{v} \in \text{null } C}_{C\vec{v} = \vec{0}}, \quad \vec{v} \not\perp f(\vec{x})$

$$\varphi(\alpha) = f(\vec{x} + \alpha\vec{v}) \approx f(x) + \alpha \underbrace{f'(x) \cdot \vec{v}}_{\neq 0}$$

ze směrů $\vec{v}, -\vec{v}$ je 1 vzestupný
1 sestupný

$$f'(x) \in (\text{null } C)^{\perp} = \text{rng } C^T$$

$$\Downarrow$$

$f'(x)$ je LK řádků C

$$\left[D_1 f(\vec{x}), \dots, D_n f(\vec{x})\right] = -\underbrace{\left[\lambda_1, \dots, \lambda_n\right]}_{\lambda} \begin{bmatrix} c_{11} & & \\ & \ddots & \\ & & c_{mn} \end{bmatrix}$$

$$\|\vec{x}\| = \vec{x}^T \vec{x}$$

$$(\|\vec{x}\|^2)' = \underbrace{\vec{x}^T + \vec{x}^T}$$

$$2\vec{x}^T$$

$$f(\vec{x}) = 1/2 \|\vec{x}\|^2 = 1/2\, \vec{x}^T \vec{x}$$

$$f'(\vec{x}) = \vec{x}^T$$

$$\vec{x} - f'(\vec{x})^T = \nabla f(\vec{x})$$

$$= (-\vec{\lambda}^T C)^T = -C^T \vec{\lambda}$$

$$C\vec{x} = \vec{d}$$

$$-\underbrace{C C^T}\ \vec{\lambda} = \vec{d}$$

$$\vec{\lambda} = -(CC^T)^{-1}\vec{d}$$

$$\vec{g}(\vec{x}) = C\vec{x} - d$$

$$\vec{g}'(\vec{x}) = C$$

$$\vec{0} = f'(\vec{x}) + \vec{\lambda}^T C = f'(\vec{x}) + \lambda g'(\vec{x})$$

$$= +C^T (CC^T)^{-1} \vec{d}$$

$$\begin{bmatrix} I_n & C^T \\ C & 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{x} \\ \vec{\lambda} \end{bmatrix} \begin{matrix} \} n \\ \} m \end{matrix} = \begin{bmatrix} \vec{0} \\ \vec{d} \end{bmatrix}$$

$$f(\vec{x}) = \tfrac{1}{2}\|A\vec{x}-\vec{b}\|^2 = \tfrac{1}{2}\underbrace{(A\vec{x}-\vec{b})^T}_{\vec{g}^T}\underbrace{(A\vec{x}-\vec{b})}_{\vec{g}} =$$

$$= f'(\vec{x}) = (A\vec{x}-\vec{b})^T \cdot A = \vec{x}^T A^T A - \vec{b}^T A = -\vec{\lambda}^T C$$

$$A^T A\vec{x} - A^T\vec{b} = -C^T\vec{\lambda}$$

$$\boxed{\begin{aligned} A^T A\vec{x} + C^T\vec{\lambda} &= A^T\vec{b} \\ C\vec{x} &= \vec{d} \end{aligned}} \qquad \begin{bmatrix} A^TA & C^T \\ C & 0 \end{bmatrix}\begin{bmatrix} \vec{x} \\ \vec{\lambda} \end{bmatrix} = \begin{bmatrix} A^T\vec{b} \\ \vec{d} \end{bmatrix}$$

$$f(\vec{x}) = \tfrac{1}{2}\left(\vec{x}^T A^T A\vec{x} - \vec{b}^T A\vec{x} - \vec{x}A\vec{b} + \vec{b}^T\vec{b}\right)$$

$$f(\vec{x}) = x^T A^T A - b^T A$$

$$\frac{\partial}{\partial x} h(\vec{x},\vec{\lambda}) = f'(\vec{x}) + \vec{\lambda}^T \cdot \vec{g}'(\vec{x})$$

$$\frac{\partial}{\partial \vec{\lambda}} h(\vec{x},\vec{\lambda}) = g(\vec{x})$$

$\min f(x,y) = x^2+2y^2$

na pimce prochodx $(0,3), (-3,0)$

A. $g(\vec{x}) = |x-y-3| = 0$ Tuky ne!

B. $g(\vec{x}) = (x-y-3)^2 = 0$ Tuky ne!

C. $g(\vec{x}) = \|x-y-3|_1 = 0$ Tuky ne!

D. $g(\vec{x}) = x-y-3 = 0$

B) $f(\vec{x}) = x^2+2y^2$

$f'(\vec{x}) = 2(x+4y)$

$$g'(x) = (2(x-y-3), -2(x-y-3))$$

$$h(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$

$$h'(x,y,\lambda) = (2x + \lambda \underbrace{2(x-y-3)}_{v}, 4y - \lambda 2\underbrace{(x-y-3)}_{v}, \underbrace{(x-y-3)^2}_{v})$$

$$0 = 2x + 2\lambda v$$

$$0 = 4y - 2\lambda v \Rightarrow \begin{matrix} x=0 \\ y=0 \end{matrix}$$

$$0 = v^2 \qquad \lambda \text{ voľné}$$

Tudy cesta nevede

D) $g'(x,y) = (1, -1)$

$$h(x,y,\lambda) = x^2 + 2y^2 + \lambda(x-y-3)$$

$$h'(x,y,\lambda) = (2x+\lambda, 4y-\lambda, x-y-3)$$

$$0 = 2x + \lambda \Rightarrow x = -\lambda/2$$

$$0 = 4y - \lambda \Rightarrow y = \lambda/4$$

$$0 = x - y - 3 \Rightarrow -\frac{\lambda}{2} - \frac{\lambda}{4} - 3 = 0$$

$$3/4\,\lambda = -3, \quad \lambda = -4$$

$x = 2$

$y = -1$