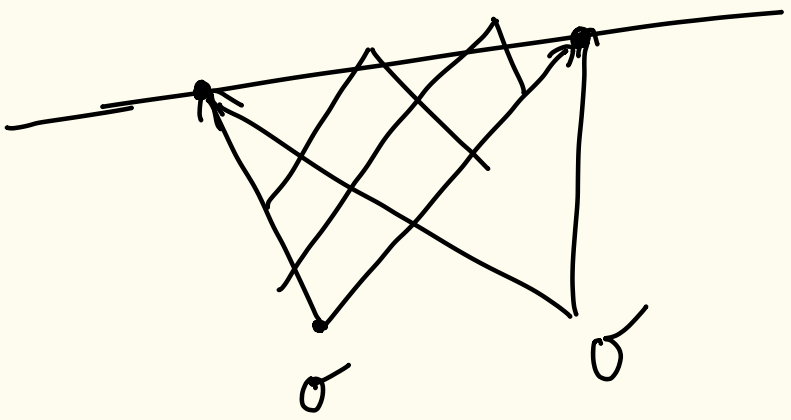
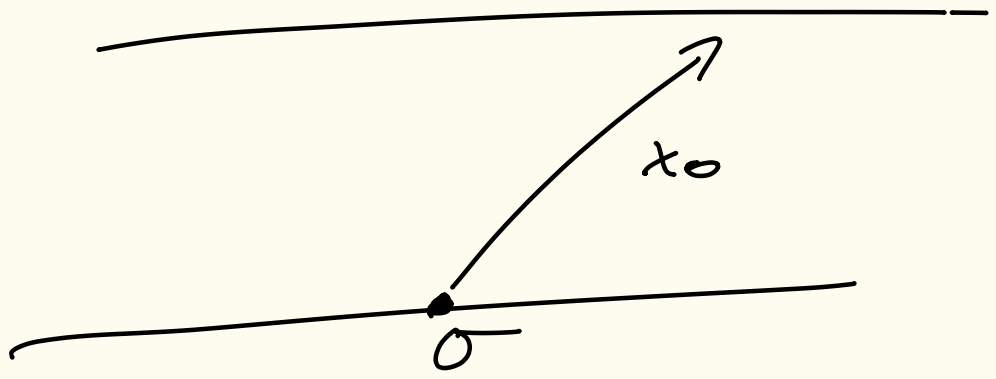




A afinní, $\overset{?}{\Rightarrow}$ x_0, X lineární

$x_0 \in A$ lib.

$X = \{X, x_0 + x \in A\}$ $x \in X \overset{?}{\Leftrightarrow} x_0 + x \in A$

$\beta_1 x_1 + \ldots + \beta_k x_k \overset{?}{\in} X$ ✓ $x_i \in X$ $\underbrace{x_0 + x_i}_{\tilde{x_i}} \in A$, $\tilde{x_i} = x_i - x_0$

$\Updownarrow$

$x_0 + (\text{—— ‖ ——}) \overset{?}{\in} A$ ✓

$x_0 + \beta_1(\tilde{x_1} - x_0) + \beta_2(\tilde{x_2} - x_0) + \ldots + \beta_k(\tilde{x_k} - x_0) \in A$

$1 \quad \beta_1 - \beta_1 \quad \beta_2 \quad -\beta_2 \quad \beta_k + \beta_k$

X lin. podprost. $\overset{?}{\Rightarrow}$ A afinní

$x_i \in A$, $\sum \alpha_i = 1$

$\alpha_1 x_1 + \ldots + \alpha_k x_k = \alpha_1(x_0 + \tilde{x_1}) + \ldots + \alpha_k(x_0 + \tilde{x_k}) =$

$= \sum_1^k \alpha_i x_0 + \alpha_1 \tilde{x_1} + \alpha_2 \tilde{x_2} + \ldots + \alpha_2 \tilde{x_k}$

$1 \quad x_0 + y \in A \qquad y \in X$

Afinní zobrazení

$A\left(\sum \alpha_i x_i\right) + b = \sum \alpha_i A x_i + \sum_1 \alpha_i \cdot b = \sum \alpha_i (A x_i + b)$

$\uparrow$

$\sum \alpha_i = 1$

f je afinní, $f(x) = g(x) + b$ $\quad b = f(0)$

$g(x)$ lineární?

$$g(x) = f(x) - f(0)$$

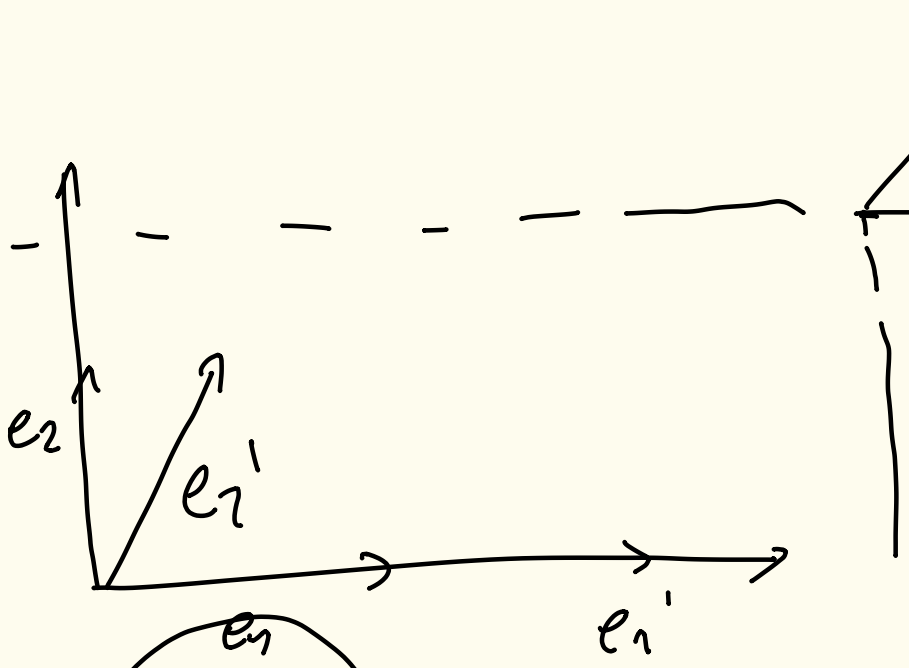
$$g\left(\sum \beta_i x_i\right) = f\left(\sum \beta_i x_i - \sum \beta_i 0 + 0\right) - f(0)$$

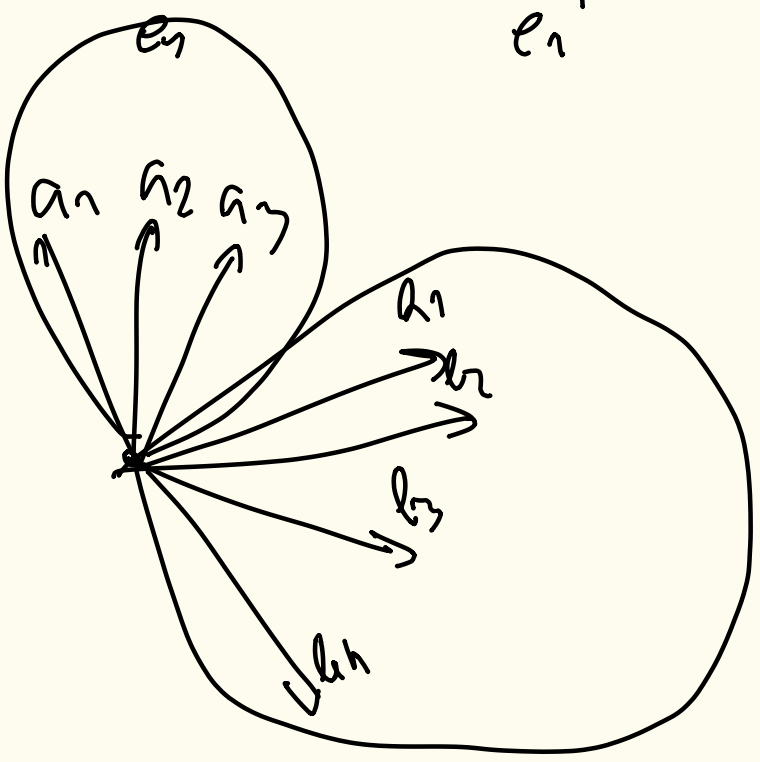
$$= \sum \beta_i f(x_i) - \sum \beta_i f(0) + f(0) - f(0)$$

$$= \sum \beta_i \underbrace{\left(f(x_i) - f(0)\right)}_{g(x_i)} = \sum \beta_i g(x_i)$$

Jiná fce ...

$$f(x_1, x_2) = \begin{bmatrix} 2 & 1/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$





$$B = \begin{bmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_h^T \end{bmatrix} \qquad Bx = 0 \qquad Ax = 0$$

$$A = \begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_h^T \end{bmatrix} \qquad Ax_0 = b$$

$$x^T y = \|x\| \|y\| \cos\varphi$$

$$U = \begin{bmatrix} | & | & | & | \\ u_1 & u_2 & \dots & u_k \end{bmatrix}$$ ← matice s ortonormálními sloupci

$$x = \alpha_1 u_1 + \alpha_2 u_2 + \dots + \alpha_k u_k$$

$$u_1^T x = \alpha_1 \cdot 1 + 0 + \dots + 0$$

$$u_1^T \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} | & | & | & | \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & & & \vdots \\ \dots & & & 1 \end{bmatrix}$$

$$U^T U = I$$

$$U^T = U^{-1} \qquad U U^{-1} = I$$

$$U U^T = I$$

$$\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

$$u \in \mathbb{R}^n \qquad \|u\| = 1$$

$$(I - 2uu^T)^T (I - 2uu^T)$$

$$(I - 2uu^T)(I - 2uu^T) = I - 4uu^T + 4u\underbrace{u^T u}_{1}u^T = I$$

$U^T U^T = U U^T$

$$f(x) = Ux$$

$$f(x)^T f(y) = x^T y$$

$$(Ux)^T Uy = x^T \boxed{U^T U}\, y = x^T I y = x^T y$$