konvexní optimalizace

$f: \mathbb{R}^n \to \mathbb{R}$ je konvex. na konvex. množině $X \subseteq \mathbb{R}^n$

Úloha konvexní optimalizace je úloha

$$\underline{\min} f(x) \text{ na } X$$

$f(x) = a^T x$

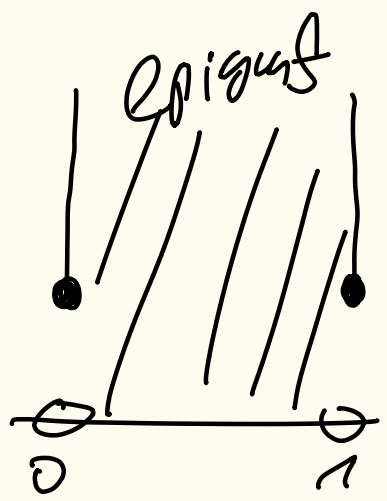
$X = \mathbb{R}^n$

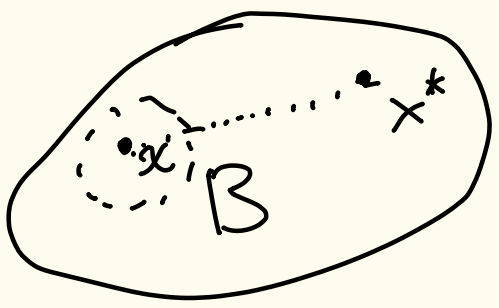
f spojitá (konvexní)

na X kompaktní konvexní

$$f(x) = \begin{cases} 0 & 0 \le x \le 1 \\ 1 & \text{jinak} \end{cases}$$

$X = [0,1]$





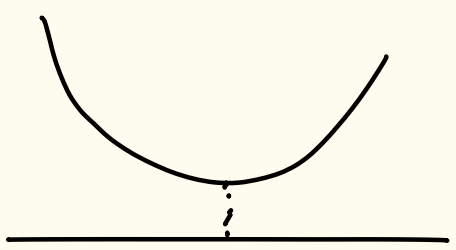
$\forall y \in B \cap X : f(x) \le f(y)$

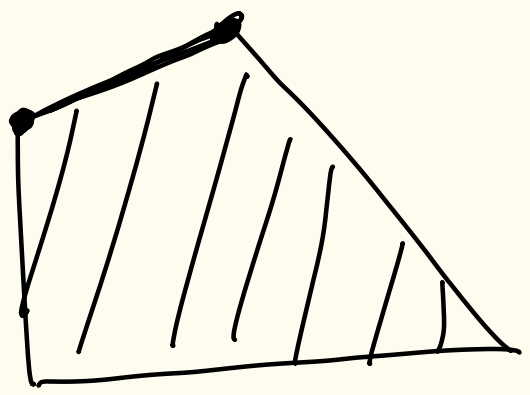
x lokální $\Rightarrow \exists x^* \in X \; f(x^*) < f(y)$

$\exists \alpha \in (0,1) \quad y = (1-\alpha)x + \alpha x^* \in B \cap X$

$f(y) \le (1-\alpha) f(x) + \alpha f(x^*)$ (<) $\rightarrow$

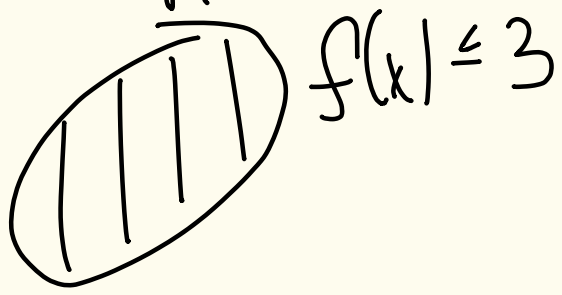
$(1-\alpha) f(x) + \alpha f(x) = f(x)$ ↯





$$\operatorname*{argmin}_{x \in X} f(x) = \{x \in X \mid f(x) \leq f(x^*)\}$$ subhladina

x^* ... optimum



ryze konvexní navíc

$$\forall \alpha \in (0,1): f((1-\alpha)x + \alpha y) < (1-\alpha) f(x) + \alpha f(y)$$

$$z = 1/2 (x+y)$$

$$f(z) = f(1/2x + 1/2y) < 1/2 f(x) + 1/2 f(y) = f(x)$$

⇓

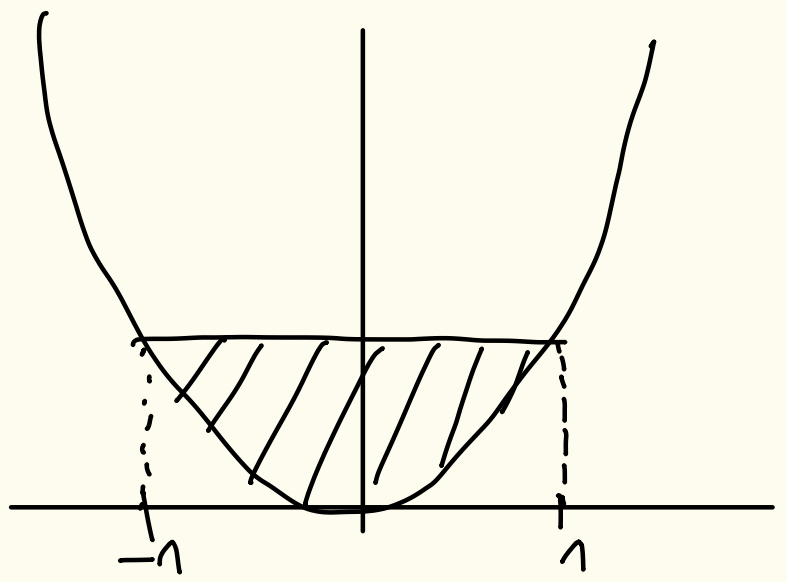
ryze konvexní fce má pouze jedno minimum

$$\min f(x)$$

z.p. $-1 \leq x_1 \leq 1$

$x_1^2 \leq x_2 \leq 1$

$x = (x_1, x_2) \in \mathbb{R}^2$



$g_1(x) = -1 - x_1$

$g_2(x) = x_1 - 1$

$g_3(x) = x_1^2 - x_2$

$g_4(x) = x_2 - 1$

$$f(x) = x^T A x + b^T x$$

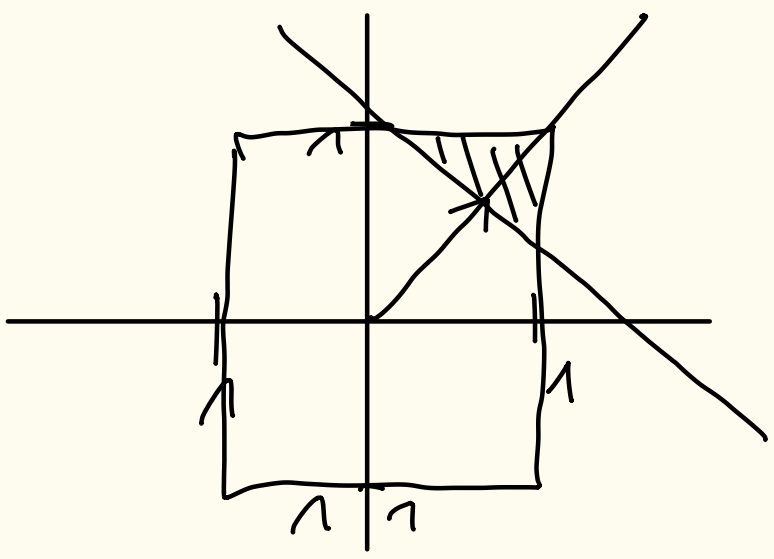
$$f'(x) = 2x^T A + b^T$$

$$f''(x) = 2A$$

$$\min \|x\|_2^2$$

$$z.p. \quad \|x\|_\infty \leq 1$$

$$x_1 + x_2\sqrt{2} \geq 0$$



$$f(x) = \|Ax-b\|^2 = x^T A^T A x + 2x^T A^T b + 2b^T b$$

Hessián $2A^TA$ → je konvex

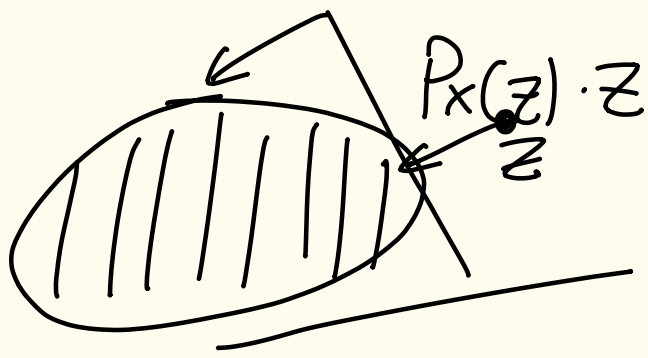
$$\min t \quad z.p. \quad \max_{i=1} \|x-a_i\| \leq t \qquad i = 1..n$$

$$t^2 \quad \dots \quad \|x-a_i\|^2 \leq t^2$$

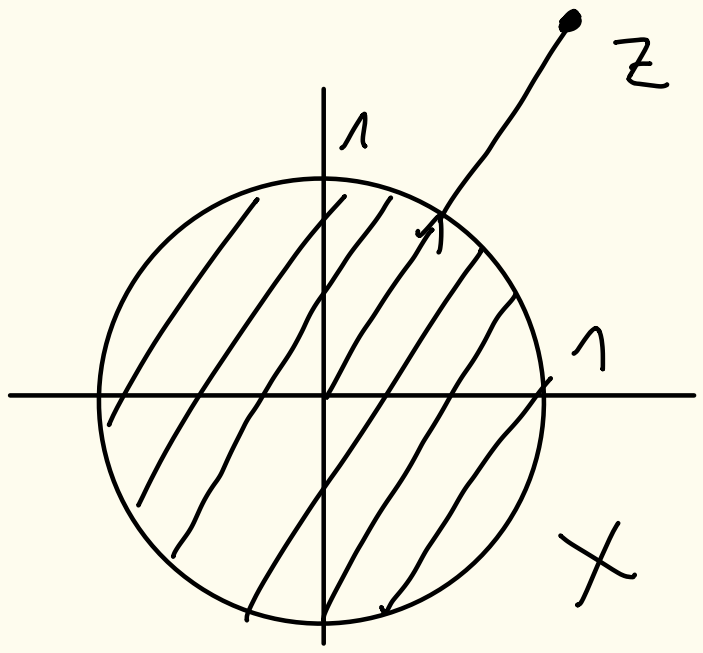
$$y \qquad \|x-a_i\|^2 \leq y$$

$$\underbrace{(P_X(z) - z)^T}_{a} x \geq \underbrace{\|P_X(z)\|^2 - z^T P_X(z)}_{b}$$

$\forall x \in X$



$x = P_X(z)$

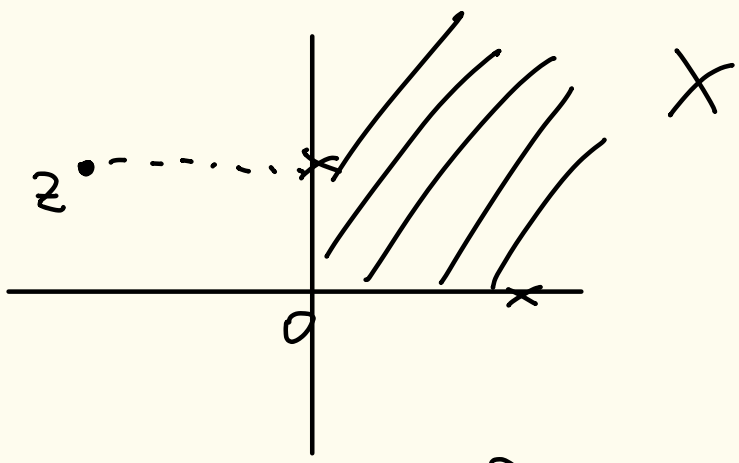


$$\left(z - \frac{z}{\|z\|}\right)^T \left(x - \frac{z}{\|z\|}\right) = z^T x - \frac{\|z\|^2}{\|z\|} - \frac{z^T x}{\|z\|} + \frac{\|z\|^2}{\|z\|^2}$$

$$= z^T x \left(1 - 1/\|z\|\right) + 1 - \|z\| \qquad \begin{matrix} \|z\| > 1 \\ \|x\| \leq 1 \end{matrix}$$

$$\leq \|z\|\|x\| \left(1 - 1/\|z\|\right) + 1 - \|z\| = \|z\|\|x\| - \|x\| + 1 - \|z\|$$

$$= \underbrace{(1 - \|z\|)}_{<0} \underbrace{(1 - \|x\|)}_{\geq 0} \leq 0$$



z

$$\left(\max(z_1, 0), \ldots, \max(z_n, 0)\right)$$