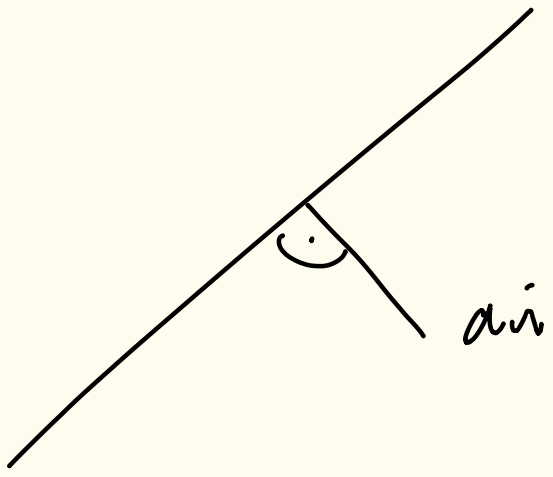
PCA jako úloha na největší stopu (Principal Component Analysis)

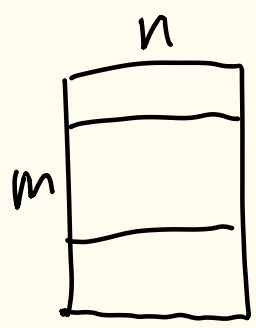


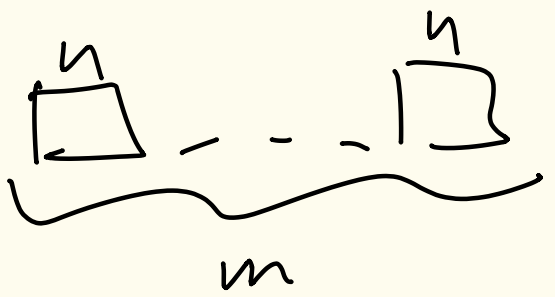
$A = V \Lambda V^T \quad tr(A) = tr(V \Lambda V^T) = tr(\Lambda \underbrace{V^T V}_{I}) = tr(\Lambda)$

$\mathbb{R}^{m \times n}$

$\mathbb{R}^n \quad \langle x, y \rangle = x^T y \quad \|x - y\|$

$\|x\| = \sqrt{x^T x}$

$\langle A, B \rangle$ 

$\Rightarrow$ 

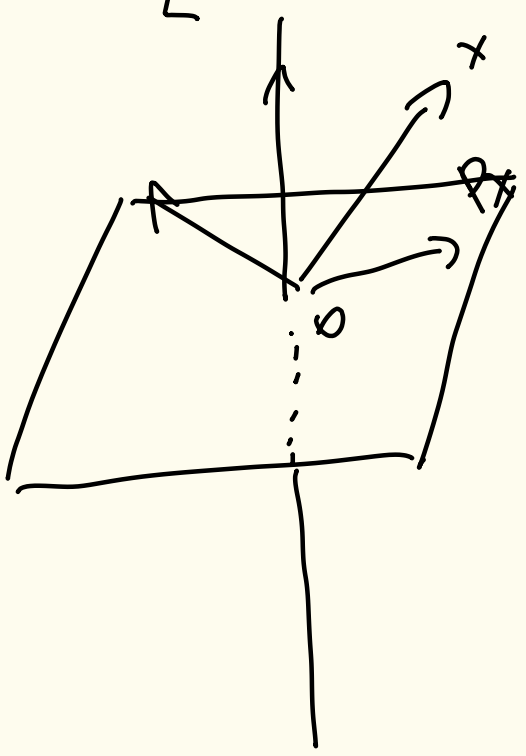
$$\langle A, B \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} b_{ij}$$

platí:

$$\langle A, B \rangle = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} b_{ij} = \sum_{j=1}^{n} \left(\sum_{i=1}^{m} a_{ij} b_{ij} \right) = \sum_{j=1}^{n} a_j^T b_j = tr(A^T B)$$

$A = [a_1 \dots a_n] \quad A^T = \begin{bmatrix} a_1^T \\ \vdots \\ a_n^T \end{bmatrix} \quad B = [b_1 \dots b_n]$

$B = [b_1 \dots b_n]$



P

$\lambda_2 = 0$

$\lambda_1 = 1$

$\|v_i v_i^T\| = tr(v_i \underbrace{v_i^T}_{\uparrow} v_i v_i^T) =$

$= tr(v_i v_i^T) = 1$

$x \in \mathbb{R}^n$

$X \in \mathbb{R}^{m \times n}$ $\quad X^T X = I$

$$\|a_i - XX^T a_i\|^2 = (a_i - XX^T a_i)^T (a_i - XX^T a_i) =$$

$$= a_i^T a_i - 2a_i^T XX^T a_i + a_i^T \underbrace{XX^T}_{\text{jednotková matice } I} XX^T a_i =$$

$\longrightarrow$ min

$$= \overbrace{\|a_i\|^2 - \underbrace{a_i^T XX^T a_i}}$$

$\longrightarrow$ max

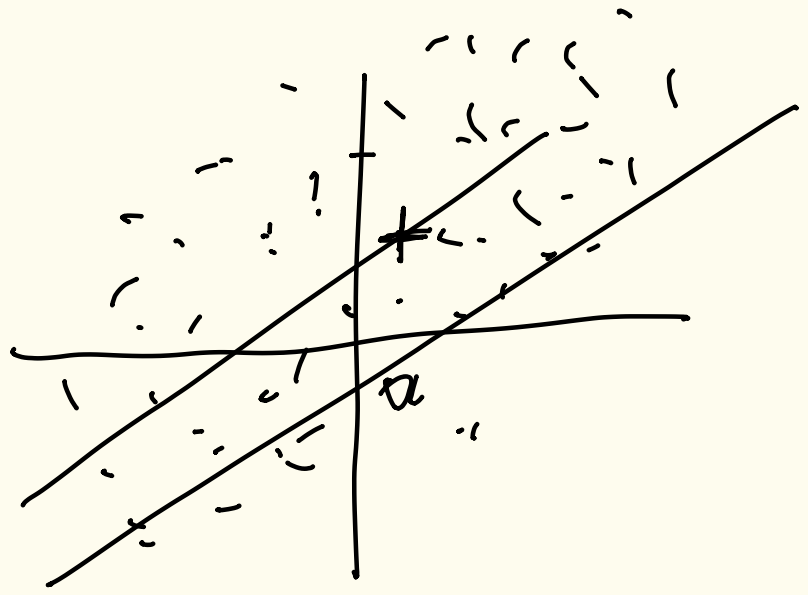
$\|X^T a_i\|$

$A = [a_1, \ldots, a_n] \in \mathbb{R}^{m \times n}$

$$tr(X^T AA^T X) = tr(\underbrace{XX^T}_{P} AA^T)$$

$$\text{Chyba} = \sum_{i=1}^{n} \|a_i\|^2 - \sum_{i=1}^{n} \|X^T a_i\|^2 \longrightarrow \lambda_1 + \ldots + \lambda_k$$

$A = [a_1 \ldots a_n]$ $\quad \|A\|^2 = tr(AA^T) = \lambda_1 + \ldots + \lambda_m$



$\overline{a}$