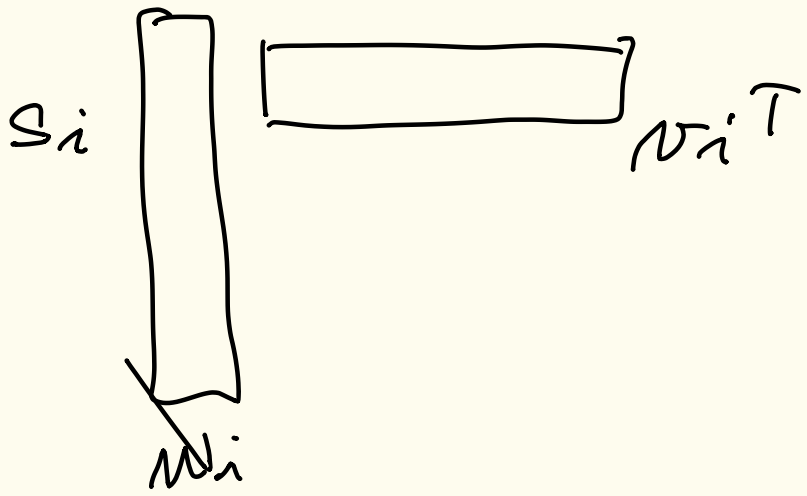


$$USV^T = [u_1 \dots u_p] \operatorname{diag}(s_1, \dots, s_p) \begin{bmatrix} v_1^T \\ \vdots \\ v_p^T \end{bmatrix}$$

$m > n$

$$\underset{m \times n}{A} = \underset{m \times n}{U} \; \underset{n \times n}{S} \; \underset{n \times n}{V^T}$$

$$= \begin{bmatrix} s_1 u_1 v_1^T \\ \vdots \\ s_p u_p v_p^T \end{bmatrix}$$

$$Av_i = USV^T v_i = USe_i = Us_i e_i =$$

(v_i underbrace e_i)

$$= s_i U e_i = s_i u_i$$

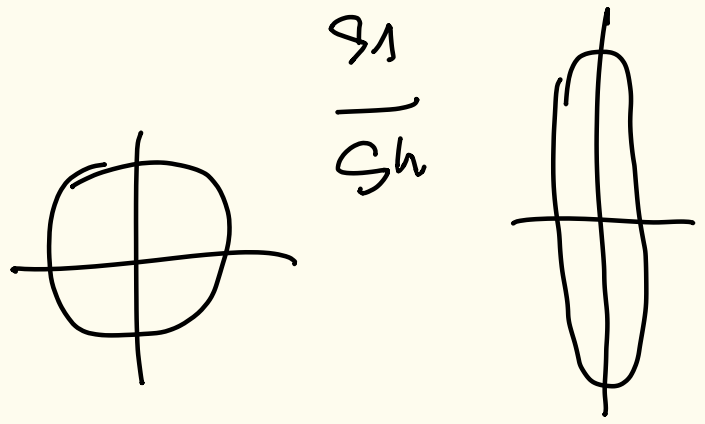
$$AA^T = USV^T VSU^T \Rightarrow AA^T = US^2U^T$$

($V^T V = I$ diag.)

$$A^T A v = \lambda v$$

$$\|A^T A v\| = \sqrt{\lambda}\,\|v\|$$

($A^T A$ underbrace v underbrace v) $\Rightarrow \lambda = 1$



$$\sum_{i=1}^{n} \|a_i - b_i\|^2 \longrightarrow MIN$$

$$Xx^T a_i \quad \| \quad \|A-B\|^2$$

$$A = [a_1 \dots a_n]$$

$$B = [b_1 \dots b_n]$$

$b_1, \dots, b_n$ leží v podprostoru $\dim = k$

$$\Leftrightarrow$$

$\underbrace{\operatorname{rank} B \le k}$

$\dim \operatorname{rng} B$

$$B^* = XX^T A \qquad AA^T = U S_k^2 U^T$$

$$U = {}_m[X \; Y]$$

$$X^T A = X^T U S V^T =$$

$$= X^T [X Y] S V^T = [X^T X \;\; X^T Y] S V^T =$$

$$= [I_k \; 0] S V^T$$

$$B^* = X [I_k \; 0] S V^T = [X \; Y] \begin{bmatrix} I_k & 0 \\ 0 & 0 \end{bmatrix} S V^T$$

$$= U S_k V^T$$

$$\|A\|^2 = tr(AA^T) = \lambda_1 + \dots + \lambda_n = s_1^2 + \dots + s_r^2$$

$$\times\ a_i \approx b_i$$

$$\|XA - B\|^2 = \langle XA - B, XA - B \rangle = \underbrace{\|XA\|^2}_{A} - 2\underbrace{\langle XA, B \rangle}_{\downarrow \text{MAX}} + \|B\|^2$$

$$\langle XA, B \rangle = tr(BA^T X^T) = tr(U S \underbrace{V^T X^T}_{R}) = tr(\underbrace{V^T X^T}_{R} U S) = \sum_i s_i \underbrace{r_{ii}}_{\leq 1}$$

(ortog)

$$|r_{ii}| \leq 1$$

$$R = I$$

$$V^T X^T U = I$$

$$\Downarrow$$

$$X = UV^T$$