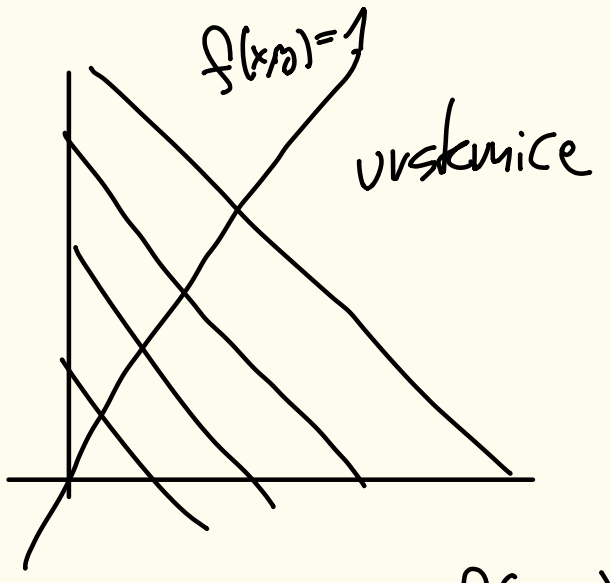
$$f(x,y) = x - 3y + 1$$

vrstvy 1

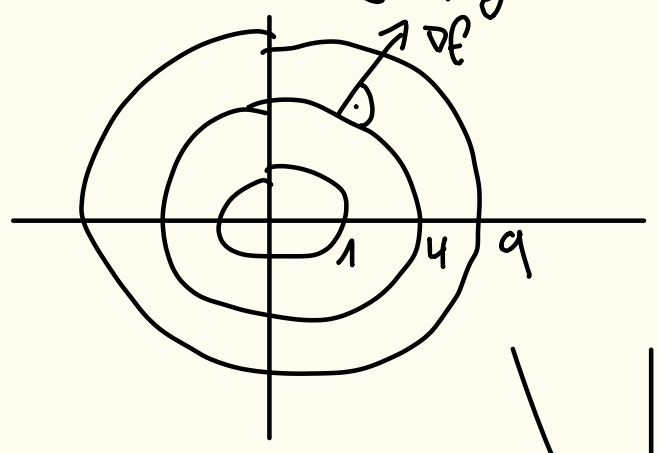
$$x - 3y + 1 = 1$$

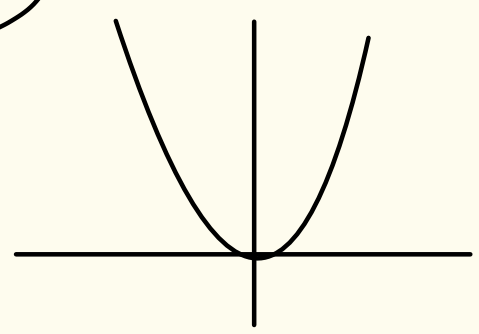


$$f'(x,y) = \left[f_x, f_y \right]$$

$$\frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y}$$

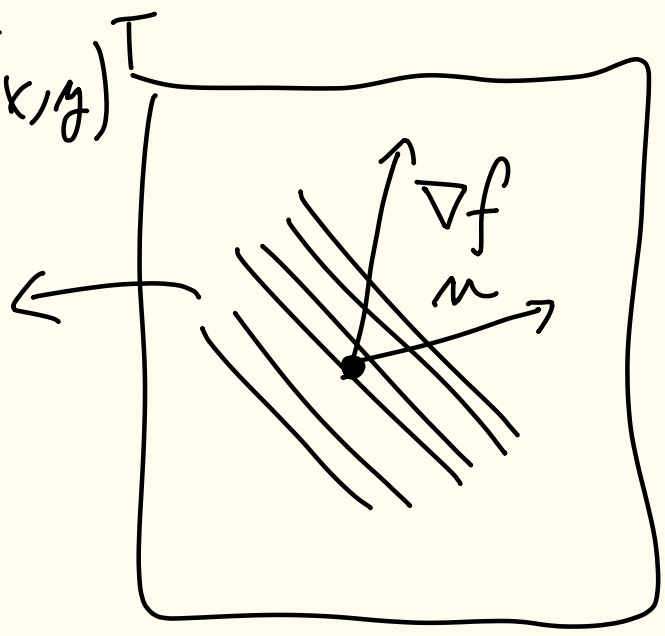
$$f(x,y) = x^2 + y^2$$





Jacobiho matice

gradient $\nabla f = f'(x,y)^T$



$\|n\| = 1$

$f_n = ?$

$f_n = \underbrace{n}_{\in \mathbb{R}^2} \cdot \underbrace{\nabla f}_{\in \mathbb{R}^2}$

$\downarrow$

$\in \mathbb{R}$

$$f_n = \lim_{\varepsilon \to 0} \frac{f(\vec{x} + \varepsilon \vec{n}) - f(\vec{x})}{\varepsilon}$$

$n = (1,0)$ směr x

$(1,0) \cdot \nabla f = f_x$

$f(x,y) = \ln(1 + xy) \qquad \mathbb{R}^2 \to \mathbb{R} \qquad x_0, y_0 = 1/2$

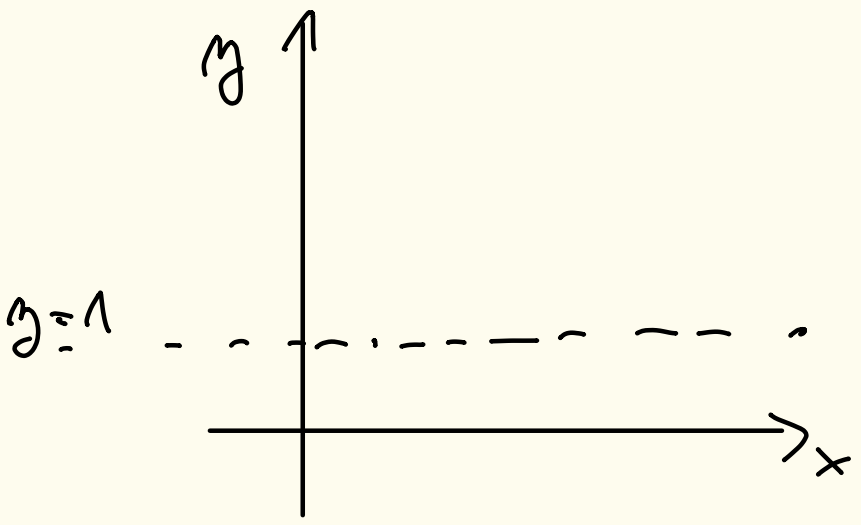
Jacobi: $\frac{\partial f}{\partial x} = \frac{y}{1+xy}$ $\qquad J = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]_{x_0, y_0} = \left[2/3, 1/3 \right]$

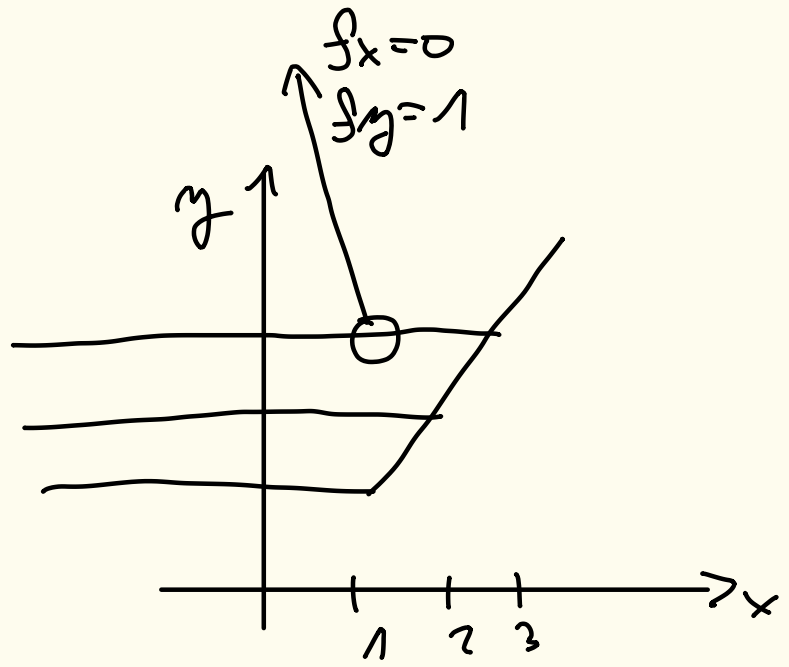
$\frac{\partial f}{\partial y} = \frac{x}{1+xy}$

Hessova matice:

$$H = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} \frac{-y^2}{(1+xy)^2} & \frac{1}{(1+xy)^2} \\ \frac{1}{(1+xy)^2} & \dots \end{bmatrix}_{x_0,y_0}$$

$f(x,y) = \max(x,y)$



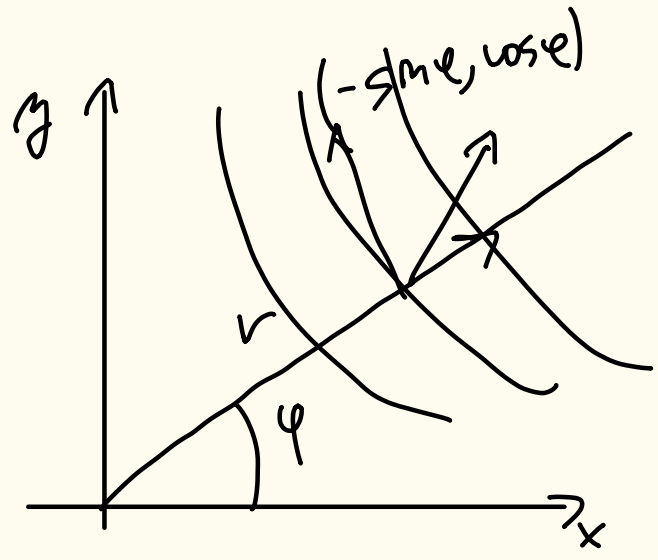


$f(x,y)$ (f_x, f_y)

→ spočti derivaci podle polárních (r, φ)

$x = r\cos\varphi$

$y = r\sin\varphi$



$$f_r = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r} =$$

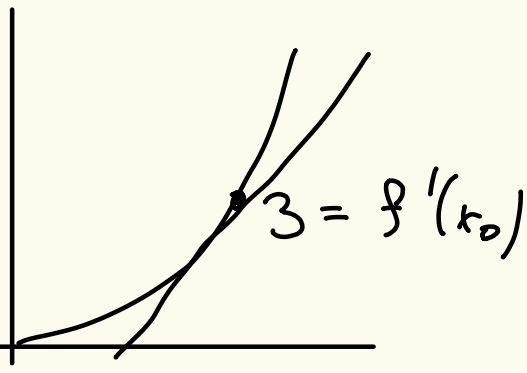
$$= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] \cdot \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \dots & \dots \end{bmatrix} = [f_x, f_y] \cdot \begin{bmatrix} \cos\varphi, \\ \sin\varphi \end{bmatrix}$$

∇f

u

$$\frac{\partial f}{\partial \varphi} = f_\varphi = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \varphi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \varphi} = \left[\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\right] \cdot$$

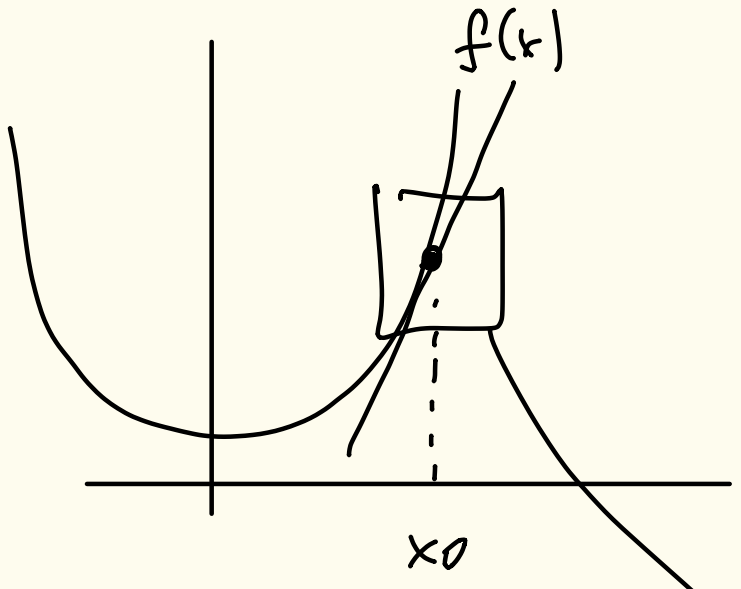
$f(\cos\varphi, \sin\varphi)$



$x = x_0 + \varepsilon$

$f(x+\varepsilon) \approx f(x_0) + 3\varepsilon$

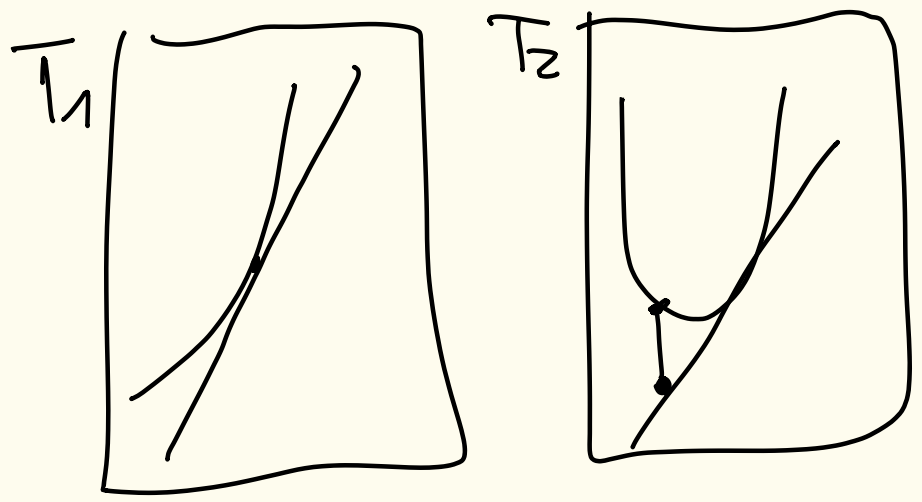
$$\begin{bmatrix} \frac{\partial x}{\partial \varphi} \cdot \frac{\partial y}{\partial \varphi} \end{bmatrix} = [f_x, f_y] \cdot [-r\sin\varphi, r\cos\varphi]$$



$$T_0(x) = f(x_0)$$

$$T_1(x) = f(x_0) + f'(x_0)(x-x_0)$$

$$+ 1/2! f''(x_0)(x-x_0)^2$$



$$f(x,y) = 6xy^2 - 2x^3 - 3y^2$$

$$(x_0, y_0) = (1, -2)$$

$$J = (f_x, f_y) = [6y^2 - 6x^2,\ 12xy - 9y^2]$$

$$J(x_0, y_0) = [18, -60]$$

$$T_0 = 24 - 2 + 24 = 26$$

$$T_1 = T_0 + [18, 60] \begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} = 46 + (18, -60) \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$$

$$= 18x - 60y + \underbrace{[46 - 18 - 120]}_{-92}$$

$$T_2 : H = \begin{bmatrix} -12x & 12y \\ 12y & 12x - 18y \end{bmatrix} \quad H(1,-2) = \begin{bmatrix} -12 & -24 \\ -24 & 48 \end{bmatrix}$$

$$T_2 = T_1 + \tfrac{1}{2}\begin{pmatrix} x-1 \\ y+2 \end{pmatrix}^T H \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$$

$$= T_1 + \begin{pmatrix} x-1 \\ y+2 \end{pmatrix} \underbrace{\tfrac{1}{2} \cdot 12}_{6} \begin{bmatrix} -1 & -2 \\ -2 & 4 \end{bmatrix} \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$$

$$= T_1 + \begin{pmatrix} x-1 \\ y+2 \end{pmatrix} 6 \cdot \begin{bmatrix} -x+1-2y-4 \\ -2x+2+4y+8 \end{bmatrix}$$

$$x^2[-6] + y^2[24] + xy[-24] + x[-12-24] + y[12+18\cdot 6]$$

$$-18x \qquad 60y$$

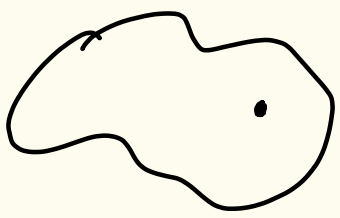
$f(x,y) = 6xy^2 - 2x^3 - 3y^3$

$(x_0, y_0) = (1, -2)$

T_2:

$-6x^2 + 24y^2 - 24xy$

$-18x + 60y + 46$



x na hranici

$R_\varepsilon(x) \subset X$

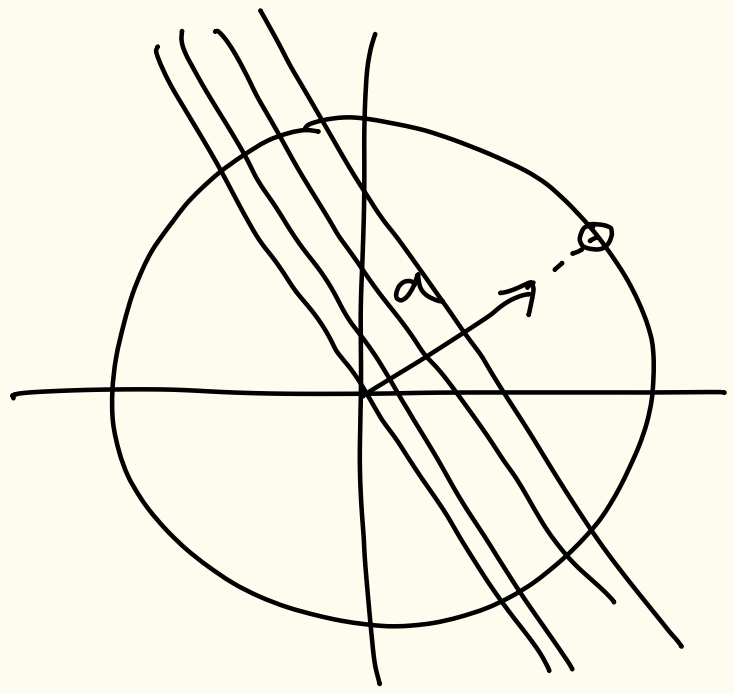
Hranice:

$\forall \varepsilon: \; R_\varepsilon(x) \cap X \neq \emptyset$

$R_\varepsilon(x) \cap \overline{X} \neq \emptyset$

$f(x) = a^T x \quad a \neq \vec{0}$

$\{ x \in \mathbb{R}^n \mid \|x\| \leq 1 \}$



sá zamk

volný extrém