$c_1 \dots c_4$

$\max \sum c_i x_i$

$-1 \le x_i \le 1$ $= \sum |c_i|$

(a) úvahou $A^T x \le b$

(b) duál

(c) komplementarity

(d) $n=3$ $c = (-2, 3, 4)$

$x_i \le 1$

b $-x_i \le 1$

A^T

(b)
$$\left[\begin{array}{ccccc} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & \\ & & \ddots & \ddots & \\ & & & & 1 \\ \hline -1 & & & & \\ & -1 & & & \\ & & \ddots & \ddots & \\ & & & & -1 \end{array}\right] \le \left[\begin{array}{c} 1 \\ \vdots \\ 1 \\ \hline 1 \\ \vdots \\ 1 \end{array}\right] d$$

x

$y_1 \ge 0$
$\vdots$
$y_n \ge 0$
$z_1 \ge 0$
$\vdots$
$z_n \ge 0$

$$\min \sum_j b_j y_j + \sum_j d_j z_j$$

Transpozice

$$\left[\begin{array}{ccc|ccc} 1 & & & -1 & & \\ & \ddots & & & \ddots & \\ & & 1 & & & -1 \end{array}\right] \begin{bmatrix} y \\ z \end{bmatrix} = c$$

$y_i - z_i = c_i$

$y_i = z_i$

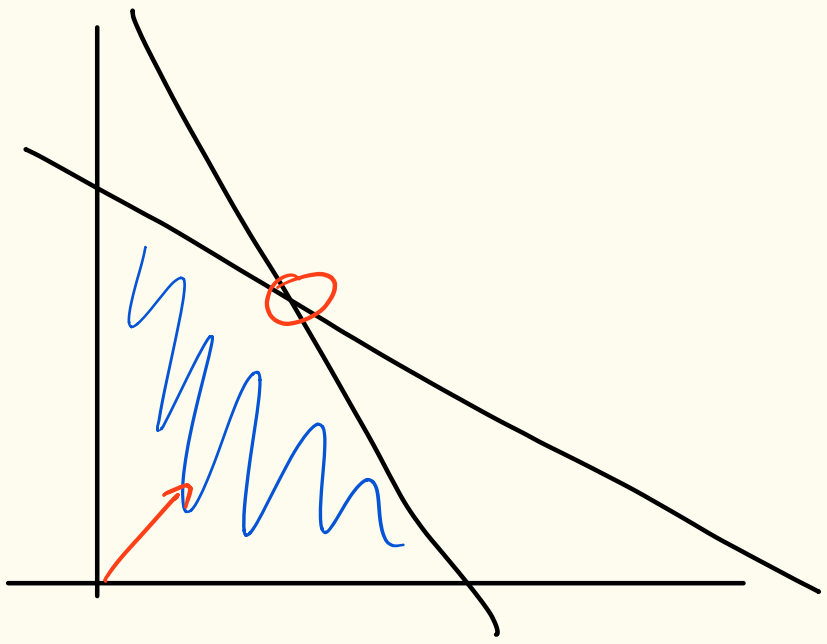
$$\min \sum_j y_j + \sum_j z_j$$

$$\max\ 50x + 40y$$

$$x + 1{,}5y \leq 750$$

$$2x + y \leq 1000$$

$$x, y \geq 0$$



$$(x_1, y_1) \ldots (x_{100}, y_{100})$$

$$f(x) = a + bx + c \sin x$$

$$\min_{a,b,c} \max_i \left| y_i - (a + bx_i + c \sin x_i) \right|$$

$$\min_{a,b,c} z$$

$$-z \leq y_i - (a + bx_i + c \sin x_i) \leq z$$

$$z \in \mathbb{R}$$

$$\min \sum |y_i - f(x_i)|$$

$$\min \sum z_i$$

$$-z_i \le y_i - f(x_i) \le z_i$$

$$\mathbb{R}^3 \to \mathbb{R}$$

f'' - vlastní čísla

H - f'' vlastní čísla

$(2,3,-1)$ -> sedlo

$(2,3,0)$ -> není max

$(2,1,1)$ -> min

$$f(x,y) = 6xy^2 - 2x^3 - 3y^4$$

$$\nabla f(x,y) = \begin{bmatrix} 6y^2 - 6x^2 \\ 12xy - 12y^3 \end{bmatrix}$$

stac. body

$$\nabla f(x) \to y^2 = x^2 \begin{cases} z = x & 12y^2(1-y) = 0 \begin{cases} y=0 \\ y=1 \end{cases} \\ z = -x & 12y^2(-1-y) = 0 \begin{cases} y=0 \\ y=-1 \end{cases} \end{cases}$$

$$f''(x) = \begin{bmatrix} -12x & 12y \\ 12y & 12x - 36y^2 \end{bmatrix} \to \begin{matrix} (0,0) \to \text{sedlo} \\ (1,1) \\ (1,-1) \end{matrix}$$

$f_1(x) = 0$

$f_2(x) = 0$

$\vdots$

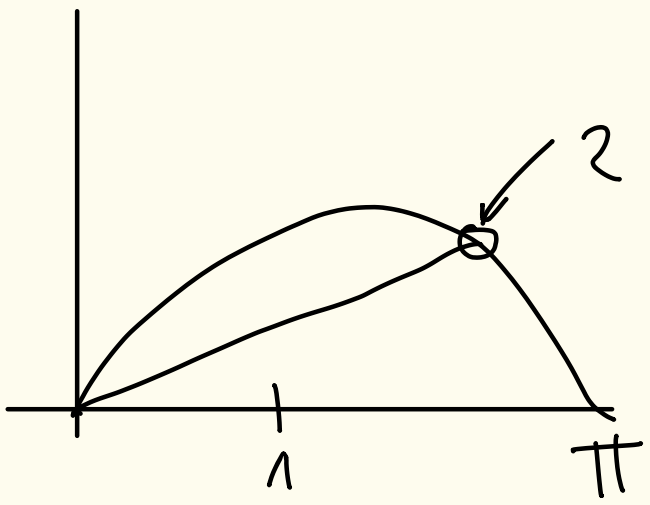
$f_n(x) = 0$

$f: \mathbb{R}^n \to \mathbb{R}^n \qquad x \in \mathbb{R}^n$

$f(x) \approx f(x_k) + f'(x_k) \cdot (x - x_k)$

$f'(x_k) \cdot (x - x_k) = -f(x_k)$ $\square$ $= 0$

$x_{k+1} = x_k - (f')^{-1} f(x_k)$



$\sin x = 1/2\, x$

$f(x) = \sin x - 1/2\, x$

$f'(x) = \cos x - 1/2$

$[f'(x)]^{-1} = 1/(\cos x - 1/2)$

minimalizace $f: \mathbb{R}^n \to \mathbb{R}$

$f': \mathbb{R}^n \to \mathbb{R}^n$

$= \vec{0}$

update

$x_{k+1} \leftarrow x_k - [f''(x_k)]^{-1} f'(x_k)$

Newton vs. Gauss Newton

↓

nelin LSQ

$$f(x) = \begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix} \qquad \min_x \sum_i f_i^2(x) = \min_x \| f(x) \|$$

$$Ax \overset{LSQ}{=} b$$

$$f(x) = f(x_k) + f'(x_k)(x - x_k)$$

$$0 = f(x_k) + J(x - x_k)$$

$$J(x - x_k) \overset{LSQ}{=} f(x_k)$$

$$x - x_k \leftarrow -(J^T J)\, J^T f(x_k)$$