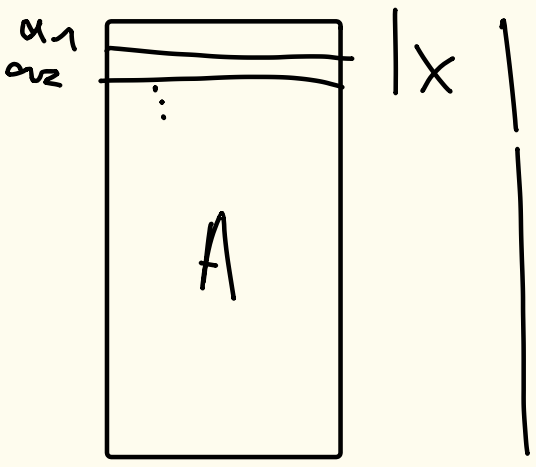
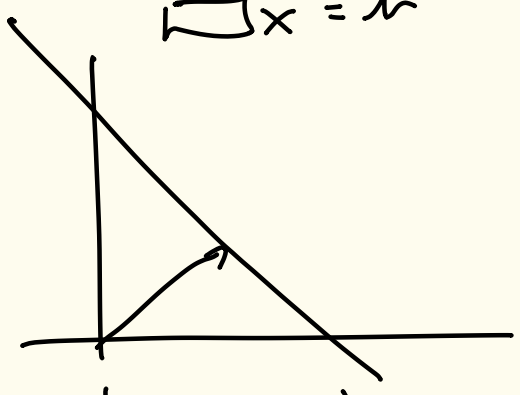


$a_1 x = b_1$
$a_2 x = b_2$

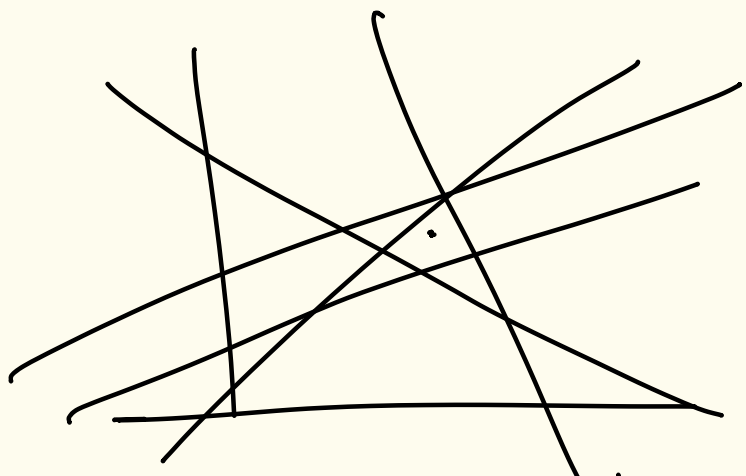
$b \in \operatorname{rng} A \Rightarrow Ax = b$

$b \in \operatorname{rng} A \Rightarrow$ najdu $x = \operatorname{argmin} \|Ax - b\|$



nelze dál soustavu

nekonečně mnoho řeš



předurčená soustava

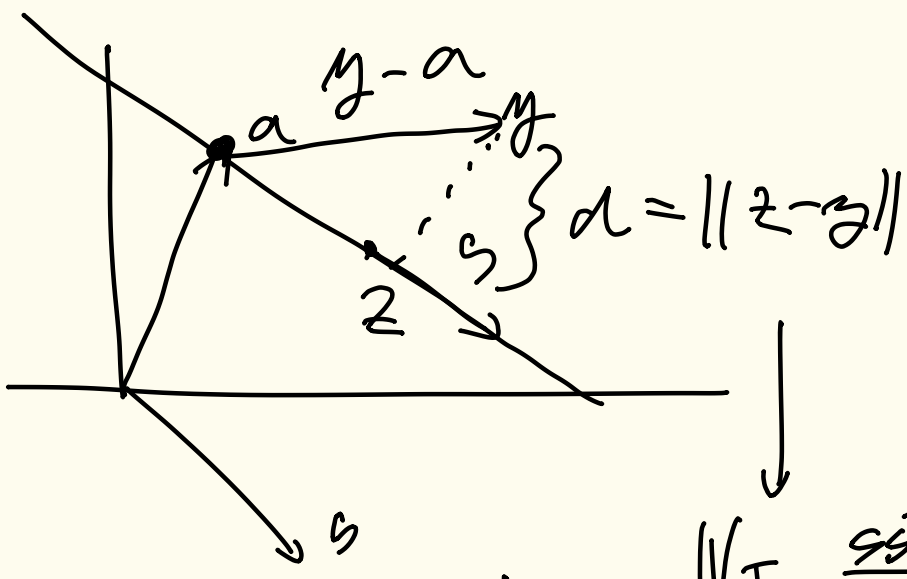
řešení neex.

$y \in \mathbb{R}^n$

$\{a + ts, t \in \mathbb{R}\}$

$a, s \in \mathbb{R}^n$

vzdálenost y od přímky



$z = a + t^* s$

$t^* = \operatorname{arg\,min} \|y - (a + ts)\|$

$d = \|z - y\|$

$Pu = q$

$u = t \quad P = s \quad q = y - a$

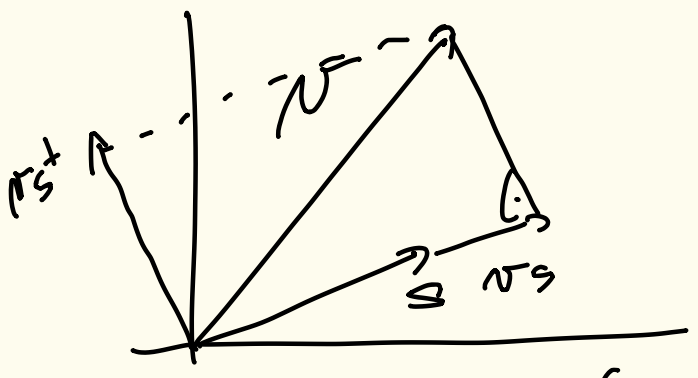
$\left\| \left(I - \frac{ss^T}{s^T s} \right)(a - y) \right\|$

$\frac{ss^T}{s^T s}$ → norma

s, c

projektor na span(s) = $\left(I - \frac{ss^T}{s^T s} \right)$ → projektor na span(s)$^\perp$

$$S \cdot t \overset{LSA}{=} y - a$$



$$v_s = \left(\frac{v \cdot s}{s \cdot s} \right) s$$

sloha $y - a$ do s

$$z = a + \frac{(y-a) \cdot s}{s \cdot s} \, s$$

$$z - y = a + \frac{(y-a) \cdot s}{s \cdot s} \, s = (a - y) - \frac{(a-y) \cdot s}{s \cdot s} \cdot s =$$

$$= \left(I - \frac{s s^T}{s^T s} \right) (a - y)$$

matice

$P = S$

$u = t$

$v = y - a$

$P u = q$

$P^T P u = P^T q$

$$u = (P^T P)^{-1} P^T q$$

$$t = (S^T S)^{-1} \cdot S^T (y - a)$$

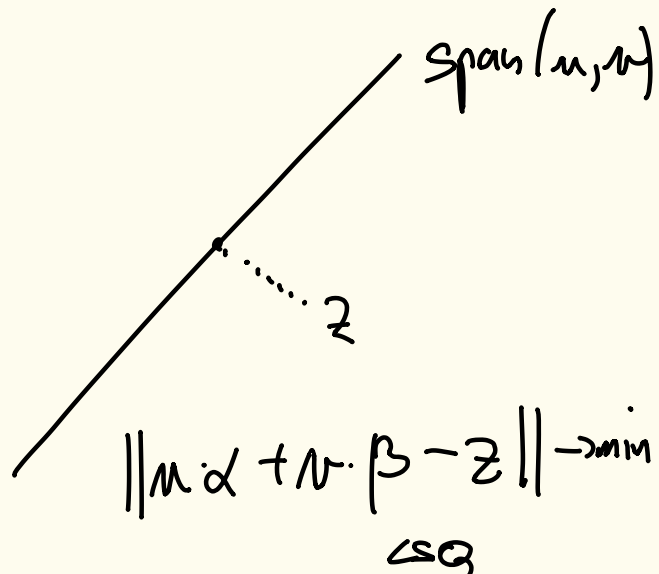
$u = (2,1,-3)$ $v = (1,-1,1)$ projekce z u na $-span(u,v)$

$-span(u,v)^{\perp}$

$z = (2,0,1)$

$A = [u, v]$ $P = A\underbrace{(A^TA)^{-1}}_{2\times 2}A^T$ $P \cdot z = 1/38\ (\dots)$

$span(u,v)^{\perp}$ → bez z = ?



$u \times v = (-2,-5,-3)$

$s = (2,5,3)$

$\frac{SS^T}{S^TS} \cdot z = \frac{7}{38}\begin{bmatrix}2\\5\\3\end{bmatrix} = 1/38\begin{pmatrix}14\\35\\21\end{pmatrix}$

$\|u\cdot\alpha + v\cdot\beta - z\| \to \min$

$u\cdot\alpha + v\cdot\beta \overset{\perp}{=} z$

$\underbrace{[u,v]}_{P}\begin{pmatrix}\alpha\\\beta\end{pmatrix} = z$

$P^TP\begin{pmatrix}\alpha\\\beta\end{pmatrix} = P^Tz$

potom $span(u,v)$

$= z - \frac{1}{38}\begin{pmatrix}14\\35\\21\end{pmatrix}$

$a = \left(-\frac{3}{5}, 0, \frac{4}{5}, 0\right)$

$b = (0,0,0,1)$

$c = (4/5, 0, 3/5, 0)$

$A = [a, b, c]$

$A^TA = I$

$A\underbrace{(A^TA)^{-1}}_{I}A^T$

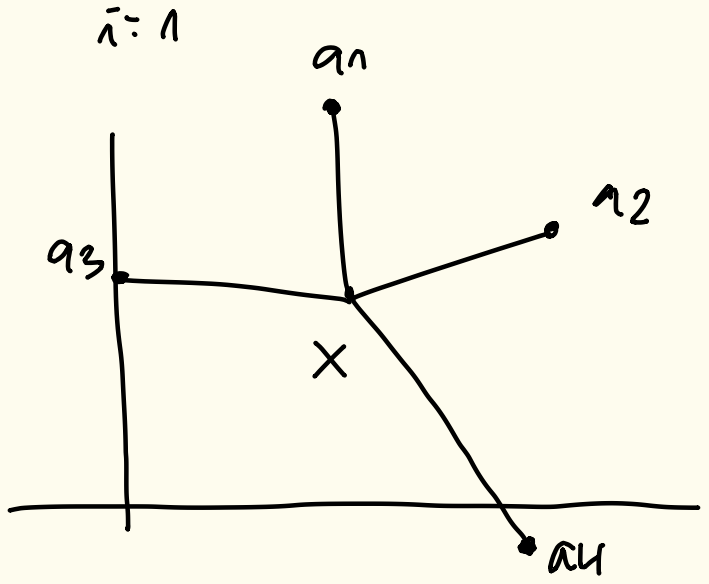
$\begin{bmatrix}1 & & \\ & 1 & \\ & 0 & 1\end{bmatrix}$

$I - [\ ] = \begin{bmatrix}0 & & \\ & 0 & \\ & 1 & 0\end{bmatrix}$

$a_1 \dots a_n \in \mathbb{R}^n$

x:

$$\sum_{i=1}^{m} \|x - a_i\|^2 \to \min$$



$$P_n = q$$

$n = x$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \\ 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

LSQ

$x = a_i \quad i = 1, \dots, m$

$P \qquad q$

$$Px = q$$

$$P^T P x = P^T q$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & 1 & \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ \vdots & \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} = m \cdot I$$

$$mIx = \begin{bmatrix} a_{1x} + a_{2x} + \cdots + a_{nx} \\ a_{1y} + a_{2y} + \cdots + a_{ny} \end{bmatrix}$$

$$mIx = \sum_{i=1}^{m} a_i \Rightarrow x = \frac{\sum a_i}{m}$$

$$\hat{R}(G,S) = G\left(a_1 + a_2 S + a_3 10^{G+S} + a_4 10^{-S}\right)$$

(R_1, G_1, S_1)

$\vdots$

(R_m, G_m, S_m)

$P_n \overset{LSQ}{=} q$

$a_1 a_2 - a_2$

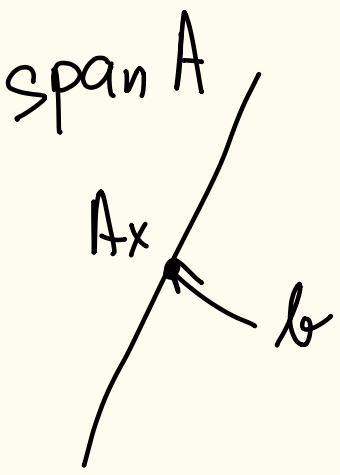
$a_1 + 10$

$$P = \begin{bmatrix} G & G\cdot S & G\cdot 10^{G+S} & G\cdot 10^{-S} \\ & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$Ax = b$$

$$(A^T A)\,x = A^T b$$

$$x = (A^T A)^{-1} A^T b$$



$\|Ax-b\|_2 \to \min$

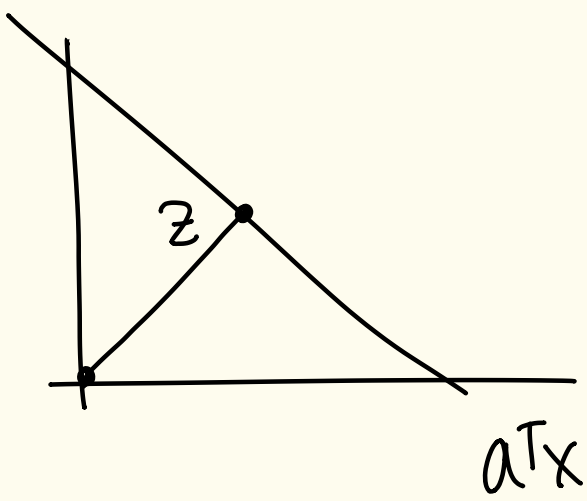
$Ax-b \perp \operatorname{span} A$

$A^T(Ax-b)=0$

5.16 DÚ

b) počátek od nadroviny $\{x \mid a^T x = b\}$

$\in \mathbb{R}^n$ V $\mathbb{R}^n$ | skalár



$z = \alpha a$, $a \perp \alpha a - b$, $\alpha = \frac{b}{\|a\|^2}$, $\|z\| = \frac{\|b\| \cdot \|a\|}{\|a\|^2}$

$z = \alpha a = \frac{ba}{\|a\|^2}$