$$\begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$$

$f(x) = \|x\|$

$x_1 \dots x_n$

$x_1^{a_1} \cdot x_2^{a_2} \dots x_n^{a_n}$

$$Av = \lambda v$$

$$(A - \lambda I)v = 0 \qquad v \neq 0$$

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ -1 & -3-\lambda \end{bmatrix}$$

$$(1-\lambda)(-3-\lambda) + 2 = 0$$

$$\begin{bmatrix} 2-\sqrt{2} & 2 \\ -1 & -2-\sqrt{2} \end{bmatrix} \sim \begin{bmatrix} 2-\sqrt{2} & 2 \\ 0 & 0 \end{bmatrix}$$

$$\lambda_{1,2} = \pm\sqrt{2} - 1$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

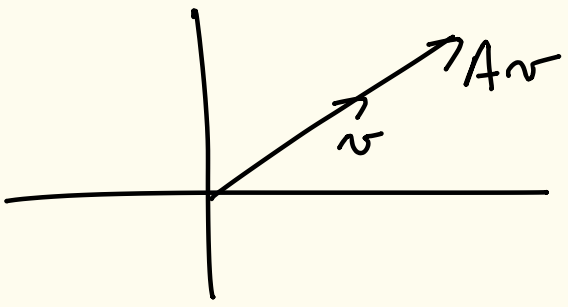
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} -\lambda & & \\ & -\lambda & \\ & & -\lambda \end{bmatrix}$$

v libovolné

$$\det(A - \lambda I) = \lambda^3 = 0$$

$$Av = \lambda v$$



$$\begin{bmatrix} 1 & & & \\ & 4 & & \\ & & 1 & \\ & & & 2 \end{bmatrix}$$

$$(1-\lambda)(4-\lambda)(1-\lambda)(2-\lambda) = 0$$

$\lambda_{1,3} = 0$
$\lambda_2 = 4$
$\lambda_4 = 2$

$\lambda = 4 \Rightarrow \begin{bmatrix} -3 & & & \\ & 0 & & \\ & & -3 & \\ & & & 2 \end{bmatrix} v = 0$

$v_2 - e_2$

$\lambda_{1,3}: \begin{bmatrix} 0 & & & \\ & 3 & & \\ & & 0 & \\ & & & 1 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$v = \alpha e_1 + \beta e_3$

$\underbrace{(A - \lambda I)}_{P} v = 0$

$\Leftrightarrow$ $\{v\}$ lin podmostor

$Pv = b$ → afinní podmostor

$$A = \begin{bmatrix} d_1 & & * \\ & d_2 & \\ 0 & & d_n \end{bmatrix} \Rightarrow A - \lambda I = \begin{bmatrix} d_1 - \lambda & & * \\ & \ddots & \\ 0 & & d_n - \lambda \end{bmatrix} \rightarrow \det = \prod_{i=1}^{n} (d_i - \lambda)$$

$\lambda \in \{d_i, i = 1 \dots n\}$

A zná λ, v
$A + \alpha I \longrightarrow$ zná λ, v

$Av = \lambda v \quad / + \alpha v$

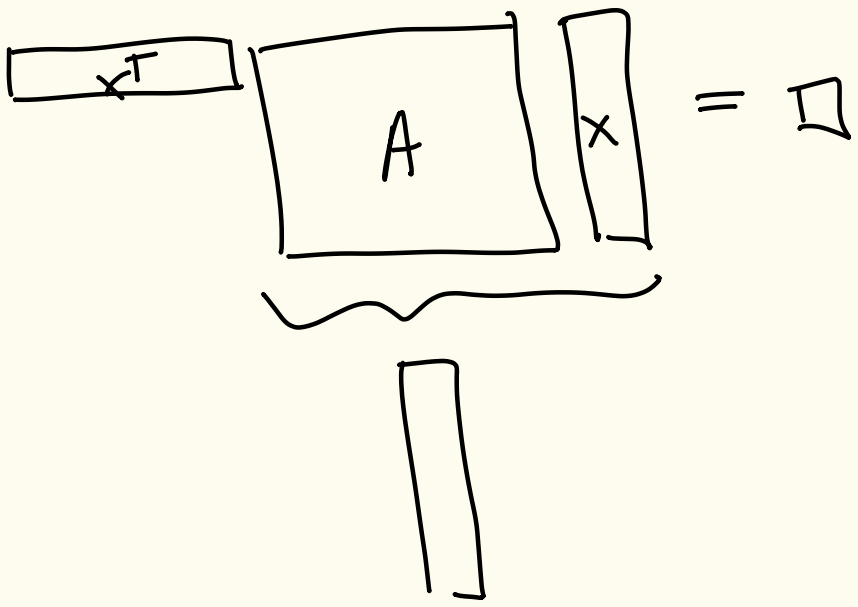
$$Av+\lambda v = \lambda v + \lambda v$$

$$\underbrace{(A+\lambda I)v}_{\lambda'} = \underbrace{(\lambda+\lambda)}_{\lambda'} v$$

v' λ' v'

$x^T A x > 0$ $\quad \forall x \neq \vec{0}$

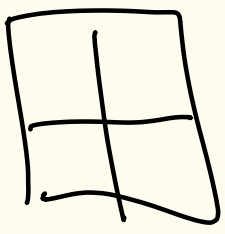
$A: A^T$ $\quad$ A pos. def



$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

$$x^T A x = (x\ y) \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x\ y) \cdot \begin{pmatrix} 2x+y \\ x+2y \end{pmatrix} =$$

$$= 2x^2 + xy + yx + 2y^2$$

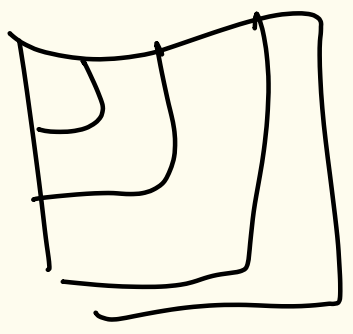


minor matice 2^n

vyber si $i \in \{1,2,3\}$

$i = \{1,3\}$

Sylvestrovo kritérium
vidět minory pos. def.



(3) $Av = \lambda v$

$$\begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

$(2-\lambda)^2 = 1$

$2-\lambda = \pm 1$

$2 \pm 1 = \lambda$

$\lambda_1 = 1$

$\lambda_2 = 3$

A symetrická

$x^T A x = \sum_i \sum_j x_i a_{ij} x_j = \sum_{ij} a_{ij} \cdot x_i x_j$

$\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \sim 2x_1^2 + 3x_2^2 - 1 x_1 x_2$

$\begin{pmatrix} 2 & -1/2 \\ -1/2 & 3 \end{pmatrix} \sim 2x_1^2 + 3x_2^2 + 1/2 x_1 x_2 - 1/2 x_1 x_2$

$A = A^T$ symetrická

$Av_1 = \lambda_1 v_1 \quad \lambda_1 \neq \lambda_2$

$Av_2 = \lambda_2 v_2 \Rightarrow v_1 \perp v_2$

$\lambda_2 v_2^T A v_1 \qquad v_2^T A \lambda_1 v_1$

$(A\lambda_2 - \lambda_1)(v_2^T A v_1) = 0$

$\neq 0 \qquad = 0$

$Av_1 = \lambda_1 v_1$

$v_2^T A v_1 - v_2^T v_1 = 0 \quad v_1 \perp v_2$

$$Av = \lambda v$$

$$A\begin{bmatrix} v_1, v_2, \dots, v_n \end{bmatrix} = \begin{bmatrix} v_1, v_2, \dots, v_n \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

V $\qquad$ V $\qquad$ Λ

$AV = V\Lambda \quad / V^T$ $\qquad$ V ... orthog. matrix

zpětná

$$AVV^T = V\Lambda V^T$$

$$\boxed{A = V\Lambda V^T}$$

A symetrická

$$x^T A x = \underbrace{x^T}_{y^T} \overbrace{V\Lambda V^T}^{A} \underbrace{x}_{y}$$

$$= y^T \Lambda y$$

$$= \lambda_1 y_1^2 + \lambda_2 y_2^2 + \dots + \lambda_n y_n^2$$