$$f(x,y) = 3x^2 + 2xy + 3y^2$$

1/2

$$= (x,y) A \begin{pmatrix} x \\ y \end{pmatrix} \qquad A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

1/2

$\rightarrow x^T A x$

$$A = \begin{bmatrix} 2 & -10 \\ 8 & 2 \end{bmatrix} \xrightarrow{\text{sym}} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$\frac{8-10}{2}$

$\downarrow$

$\frac{1}{2}(A + A^T)$

$= a u^2 + b v^2$ $\qquad \begin{bmatrix} u \\ v \end{bmatrix} = U \begin{bmatrix} x \\ y \end{bmatrix}$ $\qquad U \dots$ ortogonální

$U^{-1} = U^T$

$A v = \lambda v$

$\vec{u} = V^T \vec{x}$

$\vec{u} = U \vec{x}$

$U = V^T$

$AV = V\Lambda$

$[v_1, \dots, v_n]$ $\qquad$ $\text{diag}(\lambda_1, \dots, \lambda_4)$

V mohu upravit tak, že je ortog

$\downarrow$ JE

$A = V\Lambda V^T \qquad V^T A V = \Lambda$

$x^T A x = \underbrace{x^T V}_{u^T} \Lambda \underbrace{V^T x}_{u}$

$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\vec{u} = \begin{bmatrix} u \\ v \end{bmatrix}$

$= u^T \Lambda u =$

$= [u\ v] \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} =$

$= \lambda_1 u^2 + \lambda_2 v^2$

$a \qquad b$

$(3-\lambda)^2 - 1 = 0$

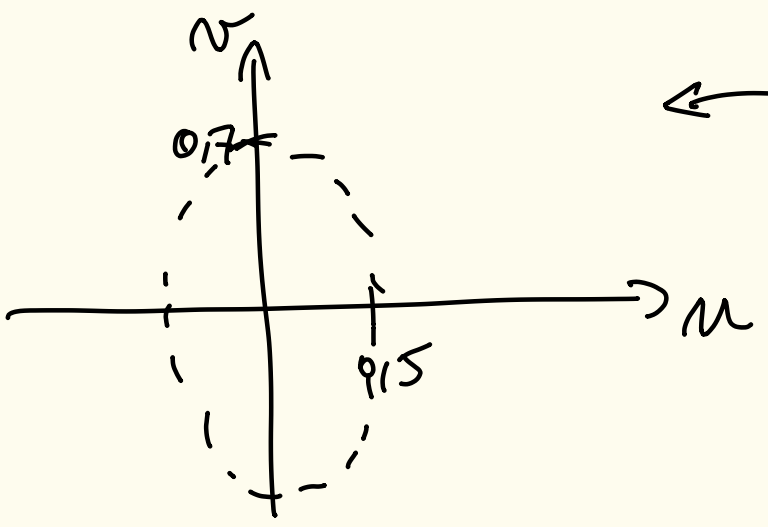
$3 - \lambda = \pm 1$

$\lambda_1 = 4 \quad \lambda_2 = 2$

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} v_1 = 0 \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} v_2 = 0$

$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$A=\begin{bmatrix}3&1\\1&3\end{bmatrix} \quad \lambda_1=4 \quad v_1=\begin{bmatrix}1\\1\end{bmatrix} \quad \lambda_2=2 \quad v_2=\begin{bmatrix}1\\-1\end{bmatrix} \quad V=1/\sqrt{2}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$$



$$au^2+bv^2=1$$

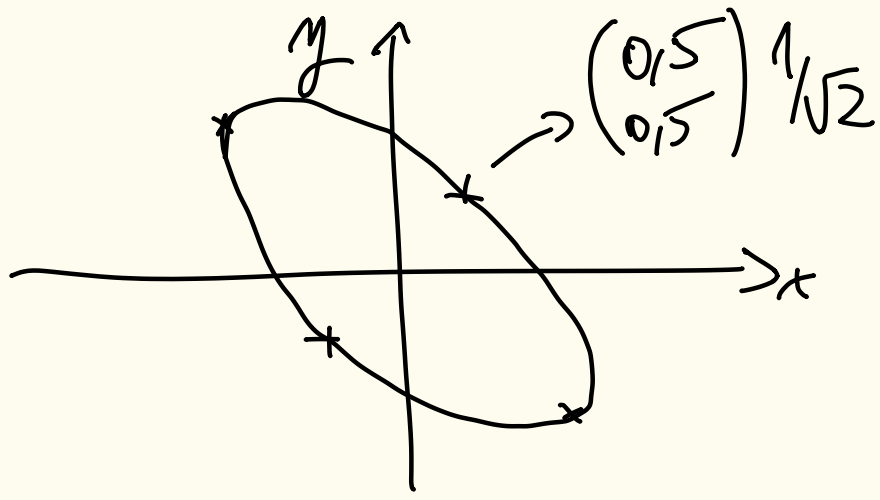
$$\leftarrow \quad 4u^2+2v^2=1$$

$$\downarrow$$

poloosy $1/\sqrt{\lambda}$

jako v předchozích (x,y)

$$\longrightarrow f(x,y)=3x^2+2xy+3y^2=1$$



Sym gauss. eliminace

sloupcem

sloupcem

$$E_2E\begin{bmatrix}2&1\\1&2\end{bmatrix}E^TE_2^T\sim\begin{bmatrix}2&1\\0&3/2\end{bmatrix}\ 1/2\ \sim\begin{bmatrix}2&0\\0&3/2\end{bmatrix}$$

neg. def.

pos def.

$\forall x\neq\vec{0}\ x^TAx>0$

neg. def.

$x^TAx<0$

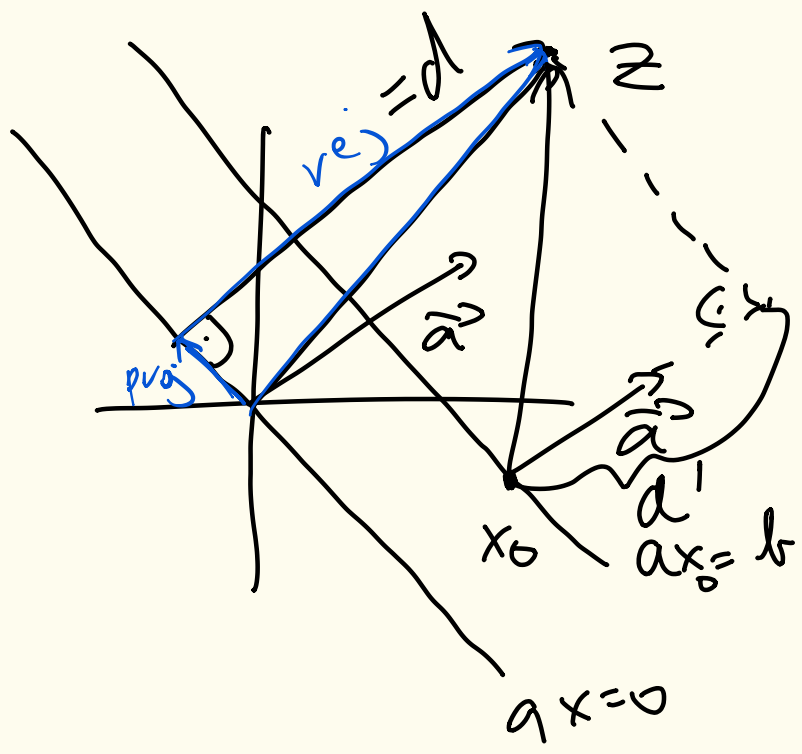
$-x^TAx>0$

$x^T(-A)x>0$

$$\{x : a^T x = b\}$$

$z, a, x \in \mathbb{R}^n$

$b \in \mathbb{R}$



$$d = \left\| \underbrace{\frac{aa^T}{a^T a}}_{P} z \right\| \qquad \frac{a \cdot z}{\|a\|}$$

$$d' = \left| \frac{a}{\|a\|} (z - x_0) \right| = \frac{|az - ax_0|}{\|a\|} = \frac{|az - b|}{\|a\|}$$

$X = \text{span}\{(1,2,3,4), (1,0,1,0), (0,1,0,0)\}$

$\rightarrow$ bázi podpr. $X^\perp$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{array}\right)$$

$2 \quad 0 \quad -2 \quad 1$

$$b = \begin{pmatrix} 2 \\ 0 \\ -2 \\ 1 \end{pmatrix} \rightarrow 1/3 \begin{pmatrix} 2 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

$P = P^T = P^2$

projekce $\rightarrow$ $P = bb^T = 1/9 \begin{bmatrix} 4 & 0 & -4 & 2 \\ 0 & 0 & 0 & 0 \\ -4 & 0 & 4 & -2 \\ 2 & 0 & -2 & 1 \end{bmatrix}$

$P : X = I - P$

$$X = (1,2,3,4) + \text{span}\left((1\,0\,1\,0), (2\,1\,0\,1), (1\,1\,1\,1) \right)$$

$$Ax = b$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$(0, -1, 0, 1)$$

$$(0, -1, 0, 1)(1, 2, 3, 4) = 2$$

$$X = \text{span}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \longrightarrow X^{\perp} = \text{span}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \quad q = 1/\sqrt{2}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

$$(q \cdot z)\, q = q q^{T} z$$