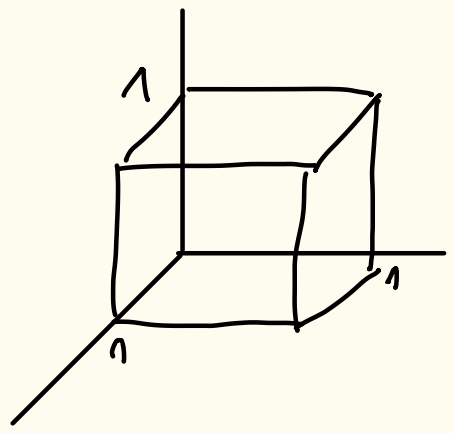
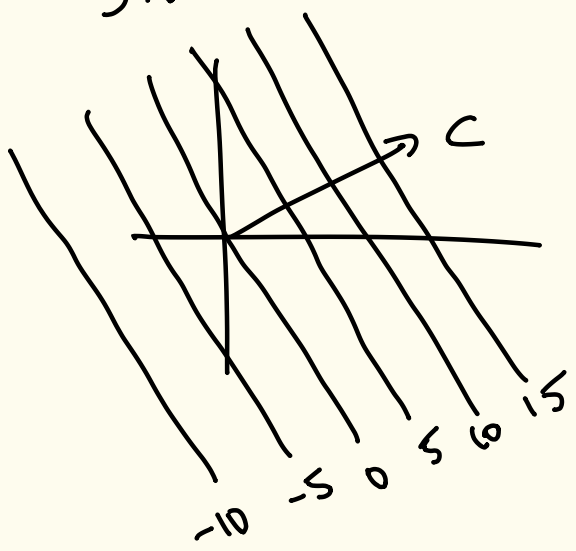
$$\max \{ c^T x, x \in \mathbb{R}^n, \vec{0} \le \vec{x} \le \vec{1} \}$$

$$0 \le x_i \le 1$$

$$i = 1 \dots n$$



$f(x) = c^T x \quad , x \in \mathbb{R}^n$



$$\max \{ c_1 x_1 + c_2 x_2 + \dots + c_n x_n \}$$

$$0 \le x_i \le 1$$

$$\max \{ c_1 x_1 \} + \max \{ c_2 x_2 \} + \dots + \max \{ c_n x_n \}$$

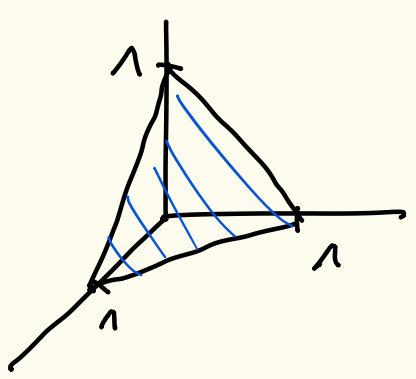
$$\max \{ c_i x_i \} \rightarrow \begin{cases} 0 & \text{pro } c_i \le 0 \\ c_i & \text{pro } c_i > 0 \end{cases}$$

$$= \sum_i \max \{ c_i x_i \}$$

$$= \sum_i \max(c_i, 0)$$

Hodnota platilo $-1 \le x_i \le 1$, protože je výsledkem $\sum_i |c_i|$

$$\max \{ c^T x, x \in \mathbb{R}^n, \quad x \ge 0, \quad x^T \mathbb{1} = 1 \}$$



$$c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

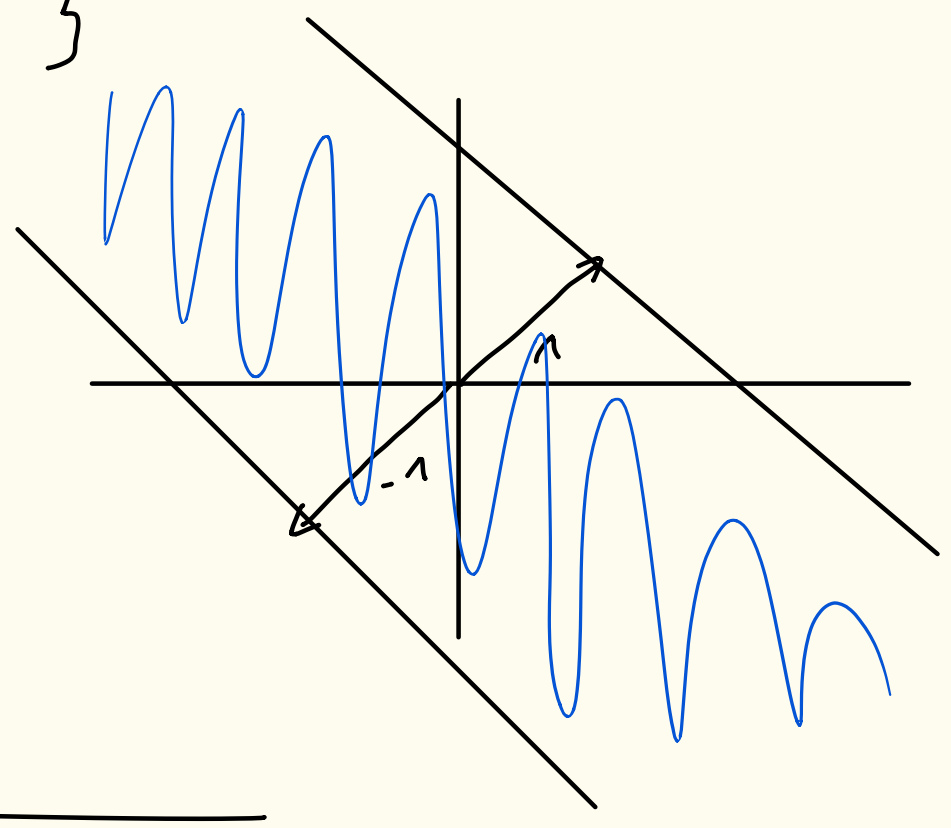
$$x_1 + x_2 + \dots + x_n = 1$$

$$x_i \ge 0$$

$$\Rightarrow \operatorname{argmax}_k c_k = i$$

$$x_i = 1 \quad x_j = 0 \quad i \ne j$$

$\max \{ c^T x, x \in \mathbb{R}^n, -1 \le x^T 1 \le 1 \}$



(1) $c(x_1 + x_2 + \dots + x_n)$

$\Rightarrow$ neomezené pokud $\exists i, j \quad c_i \neq c_j$

(2) omezené pokud

...

$\min \; 2x_1 - 3x_3 + x_4$

$x_1 - x_2 - x_3 \ge 0$

$-x_1 + 2x_2 - 3x_3 \le 5$

$2x_1 - x_2 - x_3 + 2x_4 = 6$

$x_1, x_2, x_3, x_4 \ge 0$

$x_1 - x_2 - x_3 - u = 0$

$-x_1 + 2x_2 - 3x_3 + v = 5$

$2x_1 - x_2 - x_3 + 2x_4 = 6$

$x_1, x_2, x_3, x_4, u, v \ge 0$

$\min r^T u$

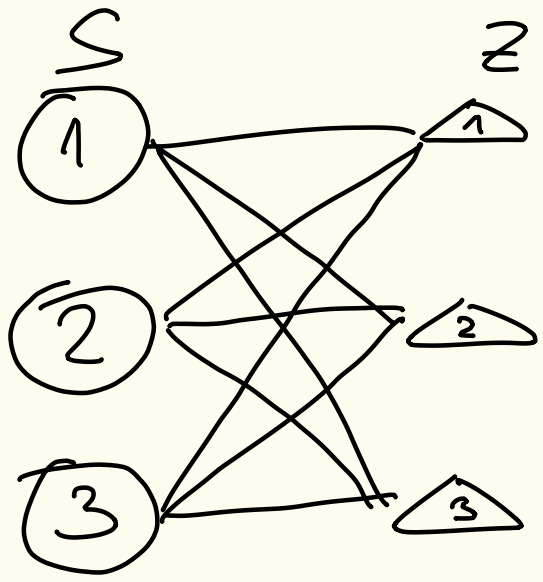
$Pu = q$

$u \ge 0$

$\vec{u} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ u \\ v \end{bmatrix}$

$\min r^T u \qquad r = \begin{bmatrix} 2 & 0 & -3 & 1 & 0 & 0 \end{bmatrix}^T$

$$\underbrace{\begin{bmatrix} 1 & -1 & -1 & 0 & -1 & 0 \\ -1 & 2 & -3 & 0 & 0 & 1 \\ 2 & -1 & -1 & 2 & 0 & 0 \end{bmatrix}}_{P} u = \underbrace{\begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}}_{q}$$



$\min \sum_{ij} c_{ij} x_{ij}$ ... $\langle c, x \rangle$ $c, x \in \mathbb{R}^{m \times n}$

$\Rightarrow x \in \mathbb{R}^{m \times n}$

$S_i \to Z_j$ x_{ij}

c_{ij} ... cena logicky jednotky zboží $i \to j$

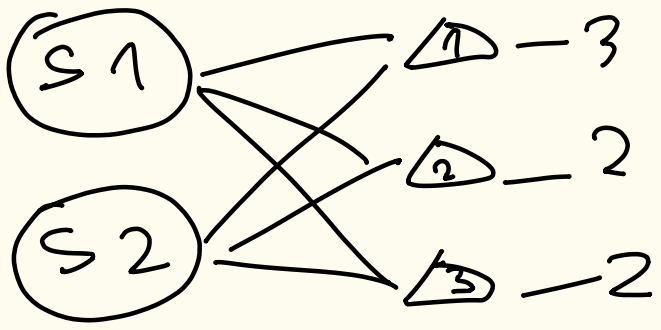
$\sum_{i=1}^{m} x_{ij} = b_j \iff X^T \mathbb{1} = b$

$\sum_{j=1}^{n} x_{ij} = a_i \iff X \mathbb{1} = a$

$x = [x_{11}, x_{12} \dots x_{1n}, x_{21}, \dots x_{mn}]^T$

$r = [c_{11}, c_{12}, \dots]^T$

$$\overbrace{\begin{bmatrix} \mathbb{1}_n & \mathbb{0}_n & \mathbb{0}_n & \dots \\ \mathbb{0}_n & \mathbb{1}_n & \mathbb{0}_n & \dots \\ \vdots & & & \\ \mathbb{0}_n & \mathbb{0}_n & \dots & \mathbb{1}_n \\ \mathbb{I}_n & \mathbb{I}_n & \dots & \mathbb{I}_n \end{bmatrix}}^{m} n = \begin{bmatrix} a_1 \\ \vdots \\ a_m \\ b \end{bmatrix}$$



$\min \max_{ij} x_{ij}$

$\Downarrow$

$\min u$

$u \geq x_{ij} \quad \forall ij$

$\sum_j x_{1j} \leq 6$ $\quad \sum_i x_{i1} = 3$

$\sum_j x_{2j} \leq 5$ $\quad \sum_i x_{i2} = 2$

$x_{ij} \geq 0$ $\quad \sum_i x_{i3} = 2$

$u \geq x_1 \qquad v \geq x_2$

$u \geq -x_1 \qquad v \geq -x_2$

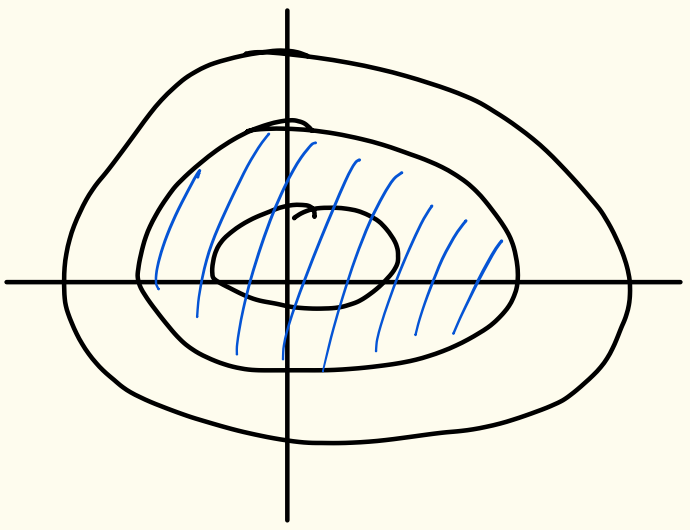
$\min(u+v)$

$\Uparrow$

$\min\{|x_1|+|x_2|\}$

$2x_1 - x_2 \geq 1$

$-x_1 + 2x_2 = 1$



$f(x,y) = x^2 + y^2$

$\{x,y;\ f(x,y) \in C\}$

$C = 1$

$x^T x$ je konvex?

$$[(\alpha-1)x + \alpha y]^T [(\alpha-1)x + \alpha y] \overset{?}{\leq} (1-\alpha)x^T x + \alpha y^T y$$

$$(1-\alpha)^2 x^T x + \alpha^2 y^T y + 2(1-\alpha)\alpha x^T y \overset{?}{\leq} (1-\alpha)x^T x + \alpha y^T y$$

$$\cos\alpha = \frac{x^T y}{\|x\|\,\|y\|} \leq 1 \Rightarrow x^T y \leq \|x\|\,\|y\|$$

$$\leq 2(1-\alpha)\alpha\|x\|\|y\| + (1-\alpha)^2\|x\|^2 + \alpha^2\|y\|^2$$

$$[(1-\alpha)\|x\| + \alpha\|y\|]^2 \overset{?}{\leq} (1-\alpha)\|x\|^2 + \alpha\|y\|^2$$

z^2 je konvex

$f(z) = z^2$

$[(1-\alpha)z + \alpha w]^2 \leq (1-\alpha)z^2 + \alpha w^2$