NEURČITÝ INTEGRÁL

$\int f(x)\,dx = F(x) + c$; $[F(x) + c]' = f(x)$, c - integračná konštanta ($c \in R$)

Ktorú funkciu ($F(x)$) musíme derivovať, aby sme dostali funkciu ($f(x)$) pod integrálom ?

$\int f(x)\,dx = F(x) + c =$ **funkcia premennej x**

základ	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$	$\int \frac{1}{x}\,dx = \ln\lvert x \rvert + c$	$\int e^x\,dx = e^x + c$	$\int a^x\,dx = \frac{a^x}{\ln(a)} + c$
$\triangle = f(x)$ $\triangle' =$ *konšt.*	$\int \sin(\triangle)\,dx = \frac{-\cos(\triangle)}{\triangle'} + c$	$\int \cos(\triangle)\,dx = \frac{\sin(\triangle)}{\triangle'} + c$	$\int e^{\triangle}\,dx = \frac{e^{\triangle}}{\triangle'} + c$	$\int a^{\triangle}\,dx = \frac{a^{\triangle}}{\triangle' \ln a} + c$
	$\int \cos(3x)\,dx = \frac{\sin(3x)}{3(=(3x)' = konšt.)} + c$ **ALE** $\int \cos(x^2)\,dx \neq \frac{\sin(x^2)}{2x\left(=(x^2)' \neq konšt.\right)} + c$			

a – konštanta

$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln(x + \sqrt{x^2 \pm a^2}) + c$ $\qquad$ $\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a} \ln\left\lvert \frac{x-a}{x+a} \right\rvert + c$

základ	**substitúcia**	
	$x \pm a = t \Rightarrow dx = dt$	$a\,x = t \Rightarrow dx = \frac{dt}{a}$, $a \neq 0$
$\int \sin(x)\,dx = -\cos(x) + c$	$\int \sin(x \pm a)\,dx = -\cos(x \pm a) + c$	$\int \sin(ax)\,dx = -\frac{\cos(ax)}{a} + c$
$\int \cos(x)\,dx = \sin(x) + c$	$\int \cos(x \pm a)\,dx = \sin(x \pm a) + c$	$\int \cos(ax)\,dx = \frac{\sin(ax)}{a} + c$
$\int \frac{1}{\cos^2(x)}\,dx = tg(x) + c$	$\int \frac{1}{\cos^2(x \pm a)}\,dx = tg(x \pm a) + c$	$\int \frac{1}{\cos^2(ax)}\,dx = \frac{tg(ax)}{a} + c$
$\int \frac{1}{\sin^2(x)}\,dx = -cotg(x) + c$	$\int \frac{1}{\sin^2(x \pm a)}\,dx = -cotg(x \pm a) + c$	$\int \frac{1}{\sin^2(ax)}\,dx = -\frac{cotg(ax)}{a} + c$
	$a\,x = t \Rightarrow dx = \frac{dt}{a}$, $a \neq 0$	$a^2 \pm x^2 = a^2\left(1 \pm \left(\frac{x}{a}\right)^2\right) = a^2(1 \pm t^2)$ $\frac{x}{a} = t \Rightarrow dx = a\,dt$, $a \neq 0$
$\int \frac{1}{\sqrt{1-(x)^2}}\,dx = \arcsin(x) + c$	$\int \frac{1}{\sqrt{1-(ax)^2}}\,dx = \frac{1}{a} \arcsin(ax) + c$	$\int \frac{1}{\sqrt{a^2-(x)^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + c$
$\int \frac{1}{\sqrt{1-(x)^2}}\,dx = -\arccos(x) + c$	$\int \frac{1}{\sqrt{1-(ax)^2}}\,dx = -\frac{1}{a} \arccos(ax) + c$	$\int \frac{1}{\sqrt{a^2-(x)^2}}\,dx = -\arccos\left(\frac{x}{a}\right) + c$
$\int \frac{1}{1+(x)^2}\,dx = arctg(x) + c$	$\int \frac{1}{1+(ax)^2}\,dx = \frac{1}{a} arctg(ax) + c$	$\int \frac{1}{a^2+(x)^2}\,dx = \frac{1}{a} arctg\left(\frac{x}{a}\right) + c$
$\int \frac{1}{1+(x)^2}\,dx = -arccotg(x) + c$	$\int \frac{1}{1+(ax)^2}\,dx = -\frac{1}{a} arccotg(ax) + c$	$\int \frac{1}{a^2+(x)^2}\,dx = -\frac{1}{a} arccotg\left(\frac{x}{a}\right) + c$

PRAVIDLÁ + METÓDY

$\int 1\,dx = x + c$, c - integračná konštanta (číslo)

$\int \textit{číslo}\,dx = \textit{číslo}\int (1)\,dx = \textit{číslo}\cdot x + c$, $\textit{číslo} \neq 0$,

$\int \textit{číslo}\cdot f(x)\,dx = \textit{číslo}\int f(x)\,dx$, konštantu (číslo) vyjmeme pred integrál

Súčet funkcií

$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

Integrál súčtu = Súčet integrálov

Rozdiel funkcií

$\int (f(x)-g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx$

Integrál rozdielu = Rozdiel integrálov

Podiel funkcií

$$\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)\,dx}{\int g(x)\,dx}$$

V čitateli je derivácia menovateľa: $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int \cot g(x)\,dx = \int \frac{\cos(x)}{\sin(x)}\,dx = \ln|\sin(x)| + c$, $\int \frac{2x}{3x^2-7}\,dx \underset{(3x^2-7)'=6x}{=} \frac{2}{6}\int \frac{6x}{3x^2-7}\,dx = \frac{2}{6}\ln|3x^2-7| + c$

Súčin funkcií

Metóda per partes

$\int (u'(x).v(x))\,dx = u(x).v(x) - \int (u(x).v'(x))\,dx$

$\int (u(x).v'(x))\,dx = u(x).v(x) - \int (u'(x).v(x))\,dx$

$\int (2x^3+1)\sin(x)\,dx = \left|\begin{array}{l}(2x^3+1) \text{ - vieme derivovať aj integrovať} \\ \sin(x) \text{ - vieme derivovať aj integrovať}\end{array}\right| =$ uvedená metóda použitá 3x

Volíme

u(x) = $(2x^3+1)$: lebo $(2x^3+1)' = 6x^2$, $(6x^2)' = 12x$, $(12x)' = 12$,

t. j. deriváciou sa znižuje mocnina premennej x a po konečnom počte derivácií dospejeme ku konštante,

v´(x) = sin(x) : derivovanie : $\sin(x)' = \cos(x)$, $\cos(x)' = -\sin(x)$

integrovanie : $\int \sin(x)\,dx = -\cos(x)$, $\int -\cos(x)\,dx = -\sin(x)$

Všimnite si **mocninu** premennej ***x*** pri derivovaní a integrovaní:

f(x)	$f'(x)$	$f''(x)$	$f'''(x) = f^{(3)}(x)$	atď.	**mocnina x**
x^6	$6x^5$	$30x^4$	$120x^3$	. . .	klesá
$\frac{1}{x^6} = x^{-6}$	$-6x^{-7} = \frac{-6}{x^7}$	$42x^{-8} = \frac{42}{x^8}$	$-336x^{-9} = \frac{-336}{x^9}$	. . .	rastie
f(x)	$\int f(x)dx$	$\int\left(\int f(x)dx\right)dx$	$\int\left(\int\left(\int f(x)dx\right)dx\right)dx$	atď.	**mocnina x**
x^6	$x^7/7$	$x^8/56$	$x^9/504$	. . .	rastie
$\frac{1}{x^6} = x^{-6}$	$\frac{x^{-5}}{-5} = \frac{1}{-5x^5}$	$\frac{x^{-4}}{20} = \frac{1}{20x^4}$	$\frac{x^{-3}}{-60} = \frac{1}{-60x^3}$	. . .	klesá

$\int 1\cdot\ln^2(x)\,dx = \left|\begin{array}{l}u = \ln^2(x) \\ v' = 1\end{array}\right| = \dots$

1 – vieme derivovať aj integrovať

$\ln^2(x)$ - vieme **len** derivovať, preto $\boldsymbol{u = \ln^2(x)}$

Súčin funkcií

Metóda substitúcie

$$\int f(g(x)).g'(x)\,dx = \left| \begin{matrix} t=g(x)\\ dt=g'(x)\,dx\end{matrix}\right| = \int f(t)\,dt = [F(t)]_{t=g(x)} + c = F[g(x)] + c$$

! Pod integrálom je funkcia ($g(x)$) a zároveň aj jej derivácia ($g'(x)$)

Príklad 1.

$$\int \frac{\sqrt{tg(x)}}{\cos^2(x)}dx = \int \sqrt{tg(x)}\cdot\frac{1}{\cos^2(x)}dx = \left| \begin{matrix} tg(x)=t \\ \frac{1}{\cos^2(x)}dx = dt, \text{lebo: } g(x)=tg(x),\ g'(x)=\frac{1}{\cos^2(x)}, \\ f(g(x))=\sqrt{(g(x))}\end{matrix}\right| = \dots$$

Možnosti g(x) pod integrálom. Keďže pod integrálom potrebujeme aj g'(x) , preto výber

g(x) :	$tg(x)$	$\sqrt{tg(x)}$	cos(x)	$\cos^2(x)$	$\frac{1}{\cos^2(x)}$
g'(x) :	$\frac{1}{\cos^2(x)}$	$\frac{1}{2}(tg(x))^{-1/2}\frac{1}{\cos^2(x)}$	$-\sin(x)$	$2\cos(x)(-\sin(x))$	$-2\cos^{-3}(x)(-\sin(x))$

Príklad 2.

Pomôcka: $(\text{číslo}\cdot g(x) \pm \text{číslo})' = \text{číslo}\cdot g'(x)$

$$\int\frac{\sin(x)}{\sqrt[5]{4\cos(x)-7}}dx = \frac{1}{-4}\int\frac{-4\sin(x)}{\sqrt[5]{4\cos(x)-7}}dx = \left|\begin{matrix} základ: g(x)=\cos(x)\\ g'(x)=-\sin(x) \\ \\ t=4\cos(x)-7 \\ dt=-4\sin(x)\,dx\end{matrix}\right| = \frac{1}{-4}\int\frac{1}{\sqrt[5]{t}}dt = \dots$$

Možnosti g(x) pod integrálom. Keďže pod integrálom potrebujeme aj g'(x) , preto výber

g(x) :	sin(x)	4cos(x)	4cos(x) –7	$\sqrt[5]{4\cos(x)-7}$
g'(x) :	cos(x)	- 4 sin(x)	- 4 sin(x)	$\frac{1}{5}(4\cos(x)-7)^{-4/5}(-4\sin(x)) = \frac{-4\sin(x)}{5\sqrt[5]{(4\cos(x)-7)^4}}$

Príklad 3.

$$\int\frac{1}{x^3}\cos\left(\frac{1}{x^2}\right)dx = \left|\begin{matrix} NIE: g(x)=\cos(1/x^2) \\ g'(x)=\sin(1/x^2)\cdot(-2x^{-3}) \\ \\ ANO: g(x)=\frac{1}{x^2}=x^{-2} \\ g'(x)=-2x^{-3}=\frac{-2}{x^3} \\ \text{-----------------} \\ t=\frac{1}{x^2} \\ dt=\frac{-2}{x^3}\end{matrix}\right| = \frac{1}{-2}\int -2\frac{1}{x^3}\cos\left(\frac{1}{x^2}\right)dx = \frac{1}{-2}\int\cos t\,dt = \dots$$

INTEGROVANIE RACIONÁLNYCH FUNKCIÍ

$\int \frac{A}{(x-a)^S}\,dx =$ A, a, S – čísla	S = 1 : $= A\int \frac{1}{x-a}\,dx = A \ln \vert x-a \vert + c$ S ≠ 1 : $\int \frac{A}{(x-a)^S}\,dx = \left\vert \begin{matrix} x-a=t \\ dx=dt \end{matrix} \right\vert = A\cdot\int t^{-S}\,dt = \dots$
$\int \frac{bx\pm c}{x^2\pm px+q}\,dx =$ b, c, p – čísla q, Q – čísla	$b=0 \ \wedge \ c=1$: $\int \frac{1}{x^2\pm px+q}\,dx \underset{Q^2=konšt.}{\overset{Q^2=\left(\pm\frac{p}{2}\right)^2+q}{=}} \int \frac{1}{\left(x\pm\frac{p}{2}\right)^2 \overset{!}{+} (Q)^2}\,dx = \left\vert \begin{matrix} x\pm\frac{p}{2}=t \\ dx=dt \end{matrix} \right\vert =$ $= \int \frac{1}{(t)^2+(Q)^2}\,dx = \frac{1}{Q^2}\int \frac{1}{\left(\frac{t}{Q}\right)^2+1}\,dt \overset{\left\vert \begin{matrix} u=\frac{t}{Q} \\ du=\frac{dt}{Q} \end{matrix} \right\vert}{=} \frac{1}{Q}\,arctg\left(\frac{u}{Q}\right)+c$ $\int \frac{1}{x^2\pm px+q}\,dx \underset{Q^2=konšt.}{\overset{Q^2=\left(\pm\frac{p}{2}\right)^2+q}{=}} \int \frac{1}{\left(x\pm\frac{p}{2}\right)^2 \overset{!}{-} (Q)^2}\,dx = \left\vert \begin{matrix} x\pm\frac{p}{2}=t \\ dx=dt \end{matrix} \right\vert =$ $= \int \frac{1}{t^2-Q^2}\,dx = \frac{1}{2Q}\ln\left\vert\frac{t-Q}{t+Q}\right\vert + c$
	$b\neq 0 \ \wedge c\neq 0$: $\int \frac{bx\pm c}{x^2\pm px+q}\,dx = \int \frac{bx\pm c}{\left(x\pm\frac{p}{2}\right)^2+Q^2}\,dx \overset{Q^2=\left(\pm\frac{p}{2}\right)^2+q}{=} \left\vert \begin{matrix} t=x\pm\frac{p}{2} \\ dt=dx \\ x=t\mp\frac{p}{2} \end{matrix} \right\vert = \int \frac{b\left(t\mp\frac{p}{2}\right)\pm c}{t^2+Q^2}\,dt \overset{A=\mp\frac{p}{2}b\pm c}{=}$ $= \int \frac{bt+A}{t^2+Q^2}\,dt = \int \frac{bt}{t^2+Q^2}\,dt + \int \frac{A}{t^2+Q^2}\,dt =$ $= b\int \frac{t}{t^2+Q^2}\,dt + A\int \frac{1}{t^2+Q^2}\,dt = \frac{b}{2}\int \frac{2t}{t^2+Q^2}\,dt + \frac{A}{Q^2}\int \frac{1}{\left(\frac{t}{Q}\right)^2+1}\,dt \overset{\left\vert \begin{matrix} u=\frac{t}{Q} \\ du=\frac{dt}{Q} \end{matrix} \right\vert}{=}$ $= \frac{b}{2}\ln\left\vert t^2+Q^2\right\vert + \frac{A}{Q}\,arctg\,\frac{u}{Q}+c$

INTEGROVANIE IRACIONÁLNYCH FUNKCIÍ

◆ **Typ funkcie:** $f\left(x, konšt., \sqrt[n_1]{\frac{ax+b}{cx+d}}, \sqrt[n_2]{\frac{ax+b}{cx+d}}, \ldots, \sqrt[n_R]{\frac{ax+b}{cx+d}}\right)$

$K=(n_1, n_2, \ldots, n_R)$ – **najmenší spoločný násobok**

volíme substitúciu : $\frac{ax+b}{cx+d}=t^K \Rightarrow \left(\frac{ax+b}{cx+d}\right)^{\frac{1}{n_i}}=t^{\frac{K}{n_i}}$, i = 1, 2, ..., R

$$\int \frac{\sqrt[6]{x}-1}{\sqrt[6]{x^7}+\sqrt[4]{x^5}}dx=\int \frac{\sqrt[6]{x}-1}{\left(\sqrt[6]{x}\right)^7+\left(\sqrt[4]{x}\right)^5}dx=\left|\begin{array}{l} K=12=(6,4) \\ x=t^{12} \\ dx=12t^{11}\,dt \\ \sqrt[6]{x}=t^{\frac{12}{6}=2},\ \sqrt[4]{x}=t^{\frac{12}{4}=3} \\ \sqrt[6]{x^7}=\left(\sqrt[6]{x}\right)^7=\left(\left(t^2\right)^7=t^{14}\right) \\ \sqrt[4]{x^5}=\left(\sqrt[4]{x}\right)^5=\left(\left(t^3\right)^5=t^{15}\right)\end{array}\right|=\int \frac{t^2-1}{t^{14}+t^{15}}12t^{11}dt=12\int \frac{t^2-1}{t^{11}\left(t^3+t^4\right)}t^{11}dt=$$

$$=12\int \frac{(t-1)\cdot(t+1)}{t^3(1+t)}dt=12\int\left(\frac{t}{t^3}-\frac{1}{t^3}\right)dt=12\int\left(t^{-2}-t^{-3}\right)dt=\ldots$$

$$\int \frac{1-\sqrt{x}}{1+\sqrt{x}}dx=\left|\begin{array}{l}\sqrt{x}=t \\ x=t^2 \\ dx=2t\,dt\end{array}\right|=\int \frac{1-t}{1+t}2t\,dt=-2\int \frac{t^2-t}{t+1}dt=\left|\left(t^2-t\right):(t+1)=\right|=-2\int\left(t-2-\frac{2}{t+1}\right)dt=\ldots$$

Pretože pre stupne polynómov $P_{m=2}(t)=\left(t^2-t\right)$ a $Q_{n=1}(t)=t+1$ platí $(m=2)\geq(n=1)$, musíme previesť delenie $P_2(t):Q_1(t)=\left(t^2-t\right):(t+1)=\ldots$ a pokračujeme vo výpočte integrálu.

◆ **Typ funkcie:**

$$f(x)=\frac{1}{\sqrt{\pm x^2\pm bx\pm c}}\rightarrow\begin{cases}\int \frac{1}{\sqrt{+x^2\pm bx\pm c}}dx=\int \frac{1}{\sqrt{(x\pm b/2)^2\pm p}}dx=\rightarrow \ln\left(x+\sqrt{x^2\pm a^2}\right) \\ \int \frac{1}{\sqrt{-x^2\pm bx\pm c}}dx=\int \frac{1}{\sqrt{p-(x\mp b/2)^2}}dx=\rightarrow \arcsin\left(\frac{x}{a}\right)\ \ alebo\ -\arccos\left(\frac{x}{a}\right)\end{cases}$$

$$\int \frac{1}{\sqrt{x^2+8x+7}}dx=\int \frac{1}{\sqrt{(x+4)^2-9}}dx=\left|\begin{array}{l}x+4=t \\ dx=dt\end{array}\right|=\int \frac{1}{\sqrt{t^2-3^2}}dt=\ln\left(t+\sqrt{t^2-3^2}\right)+c=\ln\left(x+\sqrt{(x+4)^2-3^2}\right)+c$$

$$\int \frac{1}{\sqrt{9+8x-x^2}}dx=\int \frac{1}{\sqrt{9-\left(x^2-8x\right)}}dx=\int \frac{1}{\sqrt{9-\left[(x-4)^2-16\right]}}dx=\int \frac{1}{\sqrt{25-(x-4)^2}}dx=\int \frac{1}{\sqrt{25\left(1-\left(\frac{x-4}{5}\right)^2\right)}}dx=$$

$$=\int \frac{1}{\sqrt{25}\cdot\sqrt{1-\left(\frac{x-4}{5}\right)^2}}dx=\left|\begin{array}{l}t=\frac{x-4}{5} \\ dt=\frac{1}{5}dx\end{array}\right|=\frac{1}{5}\int \frac{1}{\sqrt{1-t^2}}5\,dt=\arcsin t+c=\arcsin\frac{x-4}{5}+c$$

◆ **Typ funkcie:** $\int \frac{1}{\sqrt[n]{+x^2\pm bx\pm c}}dx=\int \frac{1}{\sqrt[n]{(x\pm d)^2}}dx=\left|\begin{array}{l}x\pm d=t \\ dx=dt\end{array}\right|=\int t^{-\frac{2}{n}}\,dt=$

INTEGROVANIE TRIGONOMETRICKÝCH FUNKCIÍ

$$\int f(\sin x).\cos x\,dx = \left| \begin{matrix} t=\sin x \\ dt=\cos x\,dx \end{matrix} \right| = \int f(t)\,dt$$

$$\int \frac{\cos x}{\sin^2 x - 6\sin x + 13}\,dx = \int \frac{\cos x}{(\sin x-3)^2+2^2}\,dx = \left|\, t=\sin x - 3 \,\right| = \int \frac{1}{t^2+2^2}\,dt = \dots$$

$$\int \frac{\cos^5 x}{\sin^3 x}\,dx = \int \frac{\cos^4 x\cdot\cos x}{\sin^3 x}\,dx = \int \frac{(1-\sin^2 x)^2\cdot \cos x}{\sin^3 x}\,dx = \left|\, t=\sin x \,\right| = \int \frac{(1-t^2)^2}{t^3}\,dt = \dots$$

$$\int f(\cos x).\sin x\,dx = \left| \begin{matrix} t=\cos x \\ dt=-\sin x\,dx \end{matrix} \right| = \int f(t)\,dt$$

$$\int (5-4\cos x)^3\cdot\sin x\,dx = \frac{1}{4}\int (5-4\cos x)^3\cdot 4\sin x\,dx = \left| \begin{matrix} t=5-4\cos x \\ dt=4\sin x\,dx \end{matrix} \right| = \frac{1}{4}\int t^3\,dt =$$

$$\int \frac{2\sin x}{\sqrt{9-\cos^2 x}}\,dx = \int \frac{2\sin x}{\sqrt{9\left(1-\left(\frac{\cos x}{3}\right)^2\right)}}\,dx = \left|\, t=\frac{\cos x}{3} \,\right| = \dots$$

$$\int \sin^5 x\cdot\cos^3\,dx = \int \sin^4 x\cdot\sin x\cdot\cos x\cdot\cos^2\,dx = \frac{1}{-2}\int (1-\cos^2 x)^2\,(-2\sin x\cdot\cos x)\cdot\cos^2\,dx = \left|\, t=\cos^2 x \,\right| =$$

$$= \frac{1}{-2}\int (1-t)^2\,t\,dt = \dots$$

Nepárny exponent

$$\int (\sin x)^{2n+1}\,dx = \int \left((\sin x)^2\right)^n.\sin x\,dx = \int (1-\cos^2 x)^n.\sin(x)\,dx = \left| \begin{matrix} t=\cos x \\ dt=\sin x\,dx \end{matrix} \right| = \int (1-t^2)^n\,dt$$

$$\int (\cos x)^{2n+1}\,dx = \int \left((\cos x)^2\right)^n.\cos x\,dx = \int (1-\sin^2 x)^n.\cos(x)\,dx = \left| \begin{matrix} t=\sin x \\ dt=\cos x\,dx \end{matrix} \right| = \int (1-t^2)^n\,dt$$

Párny exponent

$\int (\sin x)^{2n}\,dx = \int (\sin^2 x)^n\,dx$ **použi vzorec** $\sin^2(kx) = \frac{1-\cos(2kx)}{2}$

napr. $\sin^4(3x) = \left(\sin^2(3x)\right)^2 = \left(\frac{1-\cos(6x)}{2}\right)^2 = \frac{1}{4}\left(1-2\cos(6x)+\frac{1+\cos(12x)}{2}\right)$

$\int (\cos x)^{2n}\,dx = \int (\cos^2 x)^n\,dx$ **použi vzorec** $\cos^2(kx) = \frac{1+\cos(2kx)}{2}$

$$\int f(\sin(x),\cos(x),\text{konšt.})\,dx = \left| \begin{matrix} t=tg\left(\frac{x}{2}\right) & ,\ \sin(x)=\frac{2t}{1+t^2} \\ x=2\,arctg(t) & ,\ \cos(x)=\frac{1-t^2}{1+t^2} \\ dx=\frac{2}{1+t^2}\,dt & \end{matrix} \right| = \dots$$

$$\int f(tg(x))\,dx = \left| \begin{array}{l} t = tg(x) \\ x = arctg(t) \\ dx = \dfrac{1}{1+t^2}\,dt \end{array} \right| = \int \frac{f(t)}{1+t^2}\,dt = \ldots$$

$$\int tg^3(x)\,dx = \left| \begin{array}{l} t = tg(x) \\ x = arctg(t) \\ dx = \dfrac{1}{1+t^2}\,dt \end{array} \right| = \int \frac{t^3}{1+t^2}\,dt = \left| \; t^3 : (t^2+1) = t - \frac{t}{t^2+1} \; \right| = \int \left(t - \frac{t}{t^2+1} \right) dt \overset{\int \frac{f'[x]}{f[x]}dx=}{=} \ldots$$

$$\int \frac{dx}{\sin^2(x)+3\cos^2(x)+2} = \int \frac{dx}{\sin^2(x)+3\cos^2(x)+2\cdot 1} = \int \frac{dx}{\sin^2(x)+3\cos^2(x)+2(\sin^2(x)+\cos^2(x))} =$$

$$= \int \frac{dx}{\cos^2(x)\left(tg^2(x)+3+2\left(tg^2(x)+1\right)\right)} = \int \frac{dx}{\cos^2(x)\left(3tg^2(x)+5\right)} = \left| \begin{array}{l} t = tg(x) \\ dx = \dfrac{1}{\cos^2(x)}\,dt \end{array} \right| = \int \frac{1}{3t^2+5}\,dt =$$

$$= \int \frac{1}{5\left(\frac{3}{5}t^2+1\right)}\,dt = \frac{1}{\sqrt{\frac{3}{5}}} \int \frac{\sqrt{\frac{3}{5}}\,dt}{5\left(\left(\sqrt{\frac{3}{5}}\,t\right)^2+1\right)} = \left| \begin{array}{l} u = \sqrt{\frac{3}{5}}\,t \\ du = \sqrt{\frac{3}{5}}\,dt \end{array} \right| = \frac{1}{5\sqrt{\frac{3}{5}}} \int \frac{1}{u^2+1}\,du = \ldots$$

NAJPOUŽÍVANEJŠIE VZORCE

$$\sin^2(x)+\cos^2(x)=1$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin^2(1x)=\frac{1-\cos(2x)}{2} \quad \Rightarrow \quad \sin(1x)=\pm\sqrt{\frac{1-\cos(2x)}{2}}$$

$$\sin^2(kx)=\frac{1-\cos(2kx)}{2} \quad \Rightarrow \quad \sin(kx)=\pm\sqrt{\frac{1-\cos(2kx)}{2}}$$

$$\cos^2(1x)=\frac{1+\cos(2x)}{2} \quad \Rightarrow \quad \cos(1x)=\pm\sqrt{\frac{1+\cos(2x)}{2}}$$

$$\cos^2(kx)=\frac{1+\cos(2kx)}{2} \quad \Rightarrow \quad \cos(kx)=\pm\sqrt{\frac{1+\cos(2kx)}{2}}$$

URČITÝ INTEGRÁL

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a) = \textbf{číslo}$$

!!! Zmena hraníc pri **substitučnej metóde** **!!!**

$$\int_1^e \frac{1+\ln(x)}{x}\,dx = \left| \begin{array}{l} 1+\ln(x) = t \\ \frac{1}{x}\,dx = dt \\ x=1 \rightarrow t=1 \ \ (=1+\ln(1)=1+0) \\ x=e \rightarrow t=2 \ \ (=1+\ln(e)=1+1) \end{array} \right| = \int_1^2 t\,dt = \left[\frac{t^2}{2}\right]_1^2 = \frac{3}{2}$$