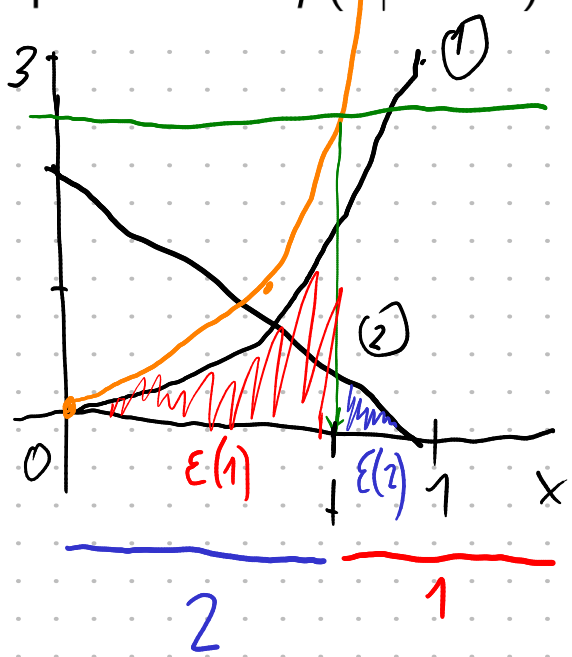
| | k=1 | k=2 |
|---|---|---|
| x=1 | 0.2 | 0.1 |
| x=2 | 0.1 | 0.3 |
| x=3 | 0.1 | 0.2 |

$$p(k|x=1) = \left[\frac{2}{3}, \frac{1}{3}\right]$$ TEST

$p(x|k=2)$

$p(x) = [0.3, 0.4, 0.3]$

pravděpodobnosti $p(x|k=1) = 3x^2$ a $p(x|k=2) = 2 - 2x$.




$$\operatorname{argmin}_q \mathcal{E}(N)$$
$$\text{s.t. } \mathcal{E}(D) \leq 0.001$$

$$\mathcal{E}(1) = \int_0^t 3x^2\,dx = \left[x^3\right]_0^t = t^3 \overset{!}{=} 0.001$$

$$t = \sqrt[3]{0.001} = 0.1$$

# MLE / MAP / Bayes

HW 1 - $p(x|k), p(k)$

HW 2 - $p(x|k)$

HW 3 - nic, jenom data $T = \{x_1, \dots, x_N\}$

chceme $N(\mu, \sigma)$

$\theta \dots \mu, \sigma$

## MLE

likelihood: $p(x|\theta)$

i.i.d - independent identically distr.b.

$$p(T|\theta) = \prod_{i=1}^{N} p(x_i|\theta)$$

nebo log-likelihood

$$l \dots \sum \log(p(x_i|\theta))$$

$$\hat{\theta} = \arg\max_{\theta} \prod_{i=1}^{N} p(x_i|\theta) \rightarrow \text{derivovat podle } \theta \text{ (tady } \mu, \sigma)$$

## MAP

$$\arg\max_{\theta} p(\theta|T) = \arg\max_{\theta} \frac{p(T|\theta) \cdot p(\theta)}{p(T)}$$

posterior

$$\hat{\theta} = \arg\max_{\theta} \prod_{i=1}^{N} p(x_i|\theta) \cdot p(\theta)$$

## Bayes

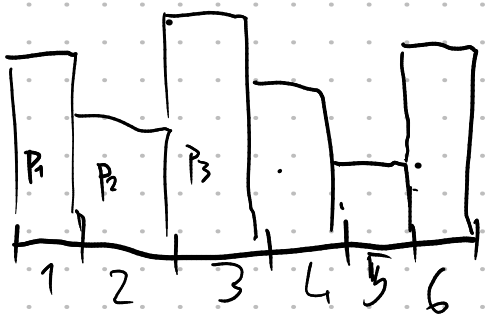
~~$\hat{\theta}$~~

$$p(x^*|T) = \int_{-\infty}^{\infty} p(x^*|\theta) \cdot p(\theta|T) \, d\theta$$

HW $\theta: \mu, \sigma$

MAP: $p(\theta)$ ... conjugate prior → Normal Inverse Gamma $\alpha, \beta, \gamma, \delta$

Discrete - categorical dist.




$$p(x=k|p_1,\dots,p_K)=p_k$$

$$\sum_i p_i = 1$$

$$L = \prod_k p_k^{N_k} \rightarrow l = \sum_{k=1}^{K} N_k \log(p_k) + \lambda\left(1-\sum_i p_i\right)$$

$$\frac{\partial l}{\partial p_1} = 0 \quad \frac{\partial l}{\partial p_2} = 0 \quad \dots \quad \frac{\partial l}{\partial \lambda} = 0$$

$$\rightarrow p_k = \frac{N_k}{\sum N_k}$$