4. (5 bodů) Vypočítejte tvar rozdělující nadplochy klasifikátoru minimalizujícího chybu klasifikace pro dvě třídy, které mají stejnou apriorní pravděpodobnost a normální rozložení $N(\mu_1, \mathbf{C}_1)$ a $N(\mu_2, \mathbf{C}_2)$ kde: $\mu_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\mathbf{C}_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ $\mathbf{C}_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$(2\pi)^{-\frac{k}{2}} \det(\boldsymbol{\Sigma})^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\top}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})},$$

$$p(x \mid k=1) > p(x \mid k=2)$$

4. (5 points) Decision tree classifiers.

(a) (2 points) Describe the simplest top-down tree-building algorithm that selects the decision rule at a node by maximization of information gain.

(b) (3 points) Discuss advantages and disadvantages of decision trees.

$$T = \{(x_i, k_i)\}_i$$

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

$$IG(A) = H(P) - \sum_{i=1}^{k} \frac{|P_i|}{|P|} H(P_i)$$

$$H(P) = -\sum_j \frac{|P^{(j)}|}{|P|} \log_2 \frac{|P^{(j)}|}{|P|} \quad \text{(entropy of } P\text{)}$$

($P^{(j)}$: number of points of class j in P)

$$H(P_i) = -\sum_j \frac{|P_i^{(j)}|}{|P_i|} \log_2 \frac{|P_i^{(j)}|}{|P_i|} \quad \text{(entropy of } P_i\text{)}$$

($P_i^{(j)}$: number of points of class j in P_i)

$$\boxed{H(P) = -\left(\frac{6}{12}\log\frac{6}{12} + \frac{6}{12}\log\frac{6}{12}\right) = -2 \times \frac{1}{2}\log_2\frac{1}{2} = 1}$$

-1

Fri $\qquad \overline{\text{Fri}}$

$5 \begin{cases} 2 \times W \\ 3 \times \overline{W} \end{cases} \qquad \begin{rcases} 4 \times W \\ 3 \times \overline{W} \end{rcases} 7$

$$H(\text{Fri}) = -\left(\frac{2}{5}\log\frac{2}{5} + \frac{3}{5}\log\frac{3}{5}\right) \qquad H(\overline{\text{Fri}}) = -\left(\frac{4}{7}\log\frac{4}{7} + \frac{3}{4}\log\frac{3}{7}\right)$$

$$IG(\text{Fri}) = 1 - \left(\frac{5}{12}H(\text{Fri}) + \frac{7}{12}H(\overline{\text{Fri}})\right)$$

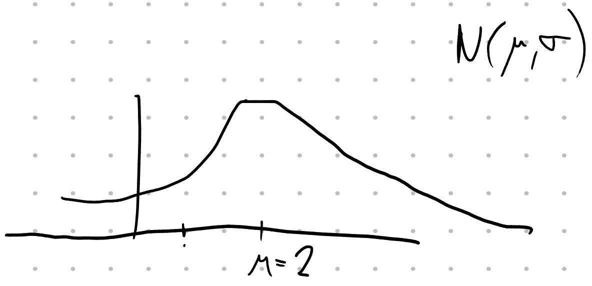
Entropie $\uparrow$ nevím nic

$\downarrow$ vím hodně

3. (5 points) Parameter Estimation. The reliability of light bulbs is defined by the probability density function

$$p(t) = \frac{1}{\theta\left(\frac{t}{\theta}+1\right)^2}, \quad t \in [0, \infty],$$

where $p(t)\delta$ is the probability that a bulb fails in a short time interval $(t, t+\delta)$. That is, the probability $P(T)$ that the bulb fails in interval $(0, T)$, is $P(T) = \int_0^T p(t)\mathrm{d}t = 1 - \frac{1}{(\frac{T}{\theta}+1)}$. What is the maximum likelihood estimate of the reliability parameter θ if in an experiment with two bulbs the following lifetimes have been observed: $t_1 = 1.0, t_2 = 4.0$? (4pts)
For this setting, what is the probability that a light bulb is still OK at time 10? (1pt)



(1) $t_1 = 1 \quad t_2 = 4.0$

MLE θ

$$\underset{\theta}{\operatorname{argmax}} \prod_i p(t_i) = \underset{\theta}{\operatorname{argmax}} \sum_i \log p(t_i)$$

$$l = \sum_i \log \frac{1}{\theta\left(\frac{t_i}{\theta}+1\right)^2} = \sum_i \log(1) - \log\left(\theta\left(\frac{t_i}{\theta}+1\right)^2\right)$$

$$\frac{\partial l}{\partial \theta} = -\sum_i \frac{1}{\cancel{\theta}\cancel{\left(\frac{t_i}{\theta}+1\right)^2}} \cdot \left(1 \cdot \cancel{\left(\frac{t_i}{\theta}+1\right)^2} + \cancel{\theta}\, 2 \cancel{\left(\frac{t_i}{\theta}+1\right)} \cdot \left(-\frac{t_i}{\theta^2}\right)\right) \overset{!}{=} 0$$

$$= -\sum_i \frac{1}{\theta} + \frac{2\left(-\frac{t_i}{\theta^2}\right)}{\frac{t_i}{\theta}+1} \overset{!}{=} 0$$

$$\theta = \dots$$

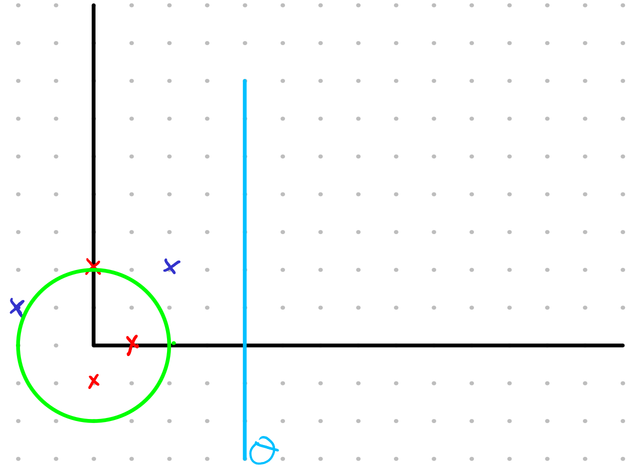
(2) $P(T) = P(t_{fail} \leq T)$

$1 - P(10)$

3. (6 bodů) Je dána trénovací množina $T = \{(\mathbf{x_i}; k_i)\}$, $i = 1, \ldots, 5$, $\mathbf{x_i} \in R^2$, $k \in \{1, -1\}$,
$T = \{(-2, 1; -1), (1, 0; 1), (0, 2; 1), (0, -1; 1), (2, 2, -1)\}$.

Algoritmem Adaboost hledáte lineární kombinaci slabých klasifikátorů.
$H(\mathbf{x}) = \sum \alpha_t h_t(\mathbf{x})$. Máte k dispozici tyto slabé klasifikátory: $h_{1\theta}(\mathbf{x}) = \delta(x_1 > \theta)$, $h_{2\theta}(\mathbf{x}) = \delta(x_1 \leq \theta)$ a $h_3(\mathbf{x}) = \delta(x_2^2 + x_1^2 < 4.5)$, $h_4(\mathbf{x}) = \delta(x_2^2 + x_1^2 \geq 4.5)$, kde $\delta()$ nabývá hodnoty 1, je-li podmínka v závorce splněna, jinak -1, θ je libovolné reálné číslo.

Nelezněte α_t.



h_3

$\varepsilon(h_3) = 0$

$$\alpha_1 = \frac{1}{2} \log\left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) = \frac{1}{2}\log\left(\frac{1}{0}\right) \to \infty$$